Congested Emergency Evacuation of a Population Using a Finite Automata Approach

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Abstract: In this paper, we address a model for population evacuation during a congested emergency event. The model employs cellular automata for space modeling and the Schadschneider model to derive the transition probabilities for the motion of the pedestrians. We describe an extension of the transition probability model that includes a component to take into account the intuitive idea that speed can be considered a direct function of population density in the modeled environment. A simulation program was encoded in C++ because of the efficiency, portability, and robustness of the programming language; the program is available from the authors upon request for educational and research purposes. A real situation was modeled and simulated with the program. All the data generated were analyzed to show the efficiency and accuracy of the new approach. Interesting new insights emerged from this analysis; notably, the results obtained are consistent with a well-known extreme value distribution.

Keywords: Finite automata; evacuation total time; extreme value distribution.

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1. Introduction

Emergency traffic computational models have drawn much attention from researchers in several areas, mainly because of the widespread availability of inexpensive and powerful computers needed to successfully address the sophistication of these models and also because of the obvious practical applications in many real-life situations. Usually, the metric used to assess the quality of a practical scenario is evacuation time (Zarboutis & Marmaras, 2007). Estimating evacuation time involves situations in which people must evacuate an environment in the shortest time possible because of natural or man-made emergency events such as hurricanes, floods, wildfires, and chemical spills.

The models that have been developed in the past to study evacuation problems use a variety of different methods, including Monte Carlo simulations (Kirchner & Schadschneider, 2002a; Smith et al., 2009; Guo & Tang, 2012), queueing (Stepanov & Smith, 2009), network theory (Cruz et al. 2005; Zheng & Liu, 2010), and hydraulic analogy (Hughes, 2002; Tian et al., 2009; Jiang et al., 2010; Li et al., 2012), being the latter very powerful and convenient for analyzing density waves in traffic flow.

Simulations can be considered in either a macroscopic or a microscopy way. Examples of the microscopy way include the multi-agent cellular automata method (Hamagmi & Hirata, 2003; Song et al., 2006; Rinaldi et al., 2007), which is the focus of this paper. Among the main advantages of the automata approach we could mention the ease of use and understanding, the possibility of simulating virtually any real environment, with and without obstacles, without the need of including complex mathematical equations (say, e.g., as for the hydrodynamical models), and the possibility of real-time visualization in the plan of the environment under study. In comparison to the hydrodynamical models, however, the main drawback is the difficulty to directly relate speed and density as a continuous function.

The main objective of this paper is to propose a new simulation model to analyze the traffic of people under emergency situations based on Schadschneider's (2002) model. The importance of this study arises from the importance of determining the best options or strategies for escape when site sheltering is not a preferred option, such as when infrastructure is damaged by hurricanes, floods, and fire.

The rest of this paper is organized as follows. In Section 2, we provide an overview of cellular automaton models, present some recent publications in the area, and introduce the proposed model. Some experiments that were performed are described and discussed in Section 3. Finally, in Section 4, we close the paper by summarizing our findings and discussing topics for future research in the area.

2. Theory

2.1. Overview

Modeling the space floor by means of cellular automata is convenient especially because of the ease with which this approach models the location of doors, corridors, and barriers, which may be crucial in an emergency evacuation. Traffic flow usually occurs in an environment (such as rooms, corridors, and stairs) in which there is only a limited amount of space available. Each individual occupies a certain area in that space, and therefore, the movement of people depends on the existence of obstacles, the density of people and the location of doors. We assume that each person is contained in a cell. According to Wolfram (1994), the idea behind these models is to consider each position (or region) of the space as a cell, which is assigned a state. The state of each cell is changed according to its state and its neighbors in the previous time step through a set of rules that attempt to mimic the biological and/or physical laws governing the real-life system. Figure 1 shows a cell in the environment and is represented by the central square. The arrows indicate all possible movement of a person in that cell. We assume that an environment can be fully distributed, so that each resulting area may contain just one person. A reasonable estimate for this area would be a square of size $\sqrt{0.2}$ meters because at a density of 5 ped/m^2, forward movement essentially comes to a halt (Tregenza, 1976). Notice that this value is somewhat in agreement with the cell size of 40 cm x 40 cm suggested by Kirchner et al. (2003).

As such, it is possible to describe the location of a person in terms of rows and columns (x, y). Therefore, we can represent an environment as an n x n array in which each position is a physical area of 0.2 m^2 that contains only one cell. This array is called an environment matrix. Each position (x, y) of this matrix has a value that is equal to 1 when filled by a person and 0 when empty. It provides an aerial view of an environment simulation of people occupying (or not) a particular area. Moving from position (x, y) to position (x, y+1) indicates that a change of state (or position) of a person has occurred, which represents his/her displacement in the environment one step further.

For purposes of simulation, another matrix called the auxiliary matrix is necessary. This matrix has the same dimensions as the environment matrix described above but only includes environmental information such as the location of doors, fixed or moving obstacles (such as chairs or tables), and physical limits (i.e., walls). A numeric code is assigned to these obstacles, and during the simulation, this information constrains the modeled persons, who obviously cannot occupy space with these obstacles. The auxiliary matrix is provided by the user through an ASCII file.

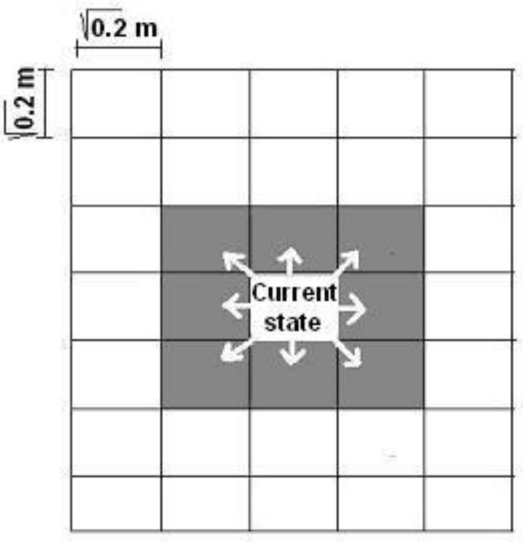


Figure 1: *Representation of a cell and its possible shifts*

To move through space, the probabilities of each person moving (or not) to a close, unoccupied neighborhood must be set. The transition probabilities have the same structure as that presented by Kirchner and Schadschneider (2002b), with several components that are used to represent the interactions between people, barriers, knowledge of the area, and the speed model (Frank & Dorso, 2009). A speed model dependent on the congestion status is a novelty introduced by the present paper into Schadschneider's (2002) model, as we will show below.

Regarding the interactions between people, the model allows for a trail to be left by other people who have followed the same path as the present person. This trail models the movement of people who are in an unknown place and would thus have to follow somebody else who already had found her/his way. For the automaton, such behavior could indicate the most visited areas, which have a higher probability of being chosen in the next step, especially for those who have not passed through this particular area. This component of the model is what we call a dynamic force because this probability is changed each time someone crosses the area. This dynamic force is more evident at the beginning and should have a steady decline because it is assumed that over time, people will have their own particular path rather than use somebody else's path. The interaction between people and obstacles in the field is called a static force because this force remains constant throughout the simulation. In fact, the static force is determined by the configuration of the environment that has been under analysis, which is considered static along time. This force is higher for the paths that lead to safe areas and smaller for areas that are close to walls, obstacles, and counter areas outside (Kirchner & Schadschneider, 2002b).

A new component of the model (and a main contribution of this paper) involves the assumption that the mean speed of a person decreases as the number of people increases and approaches the capacity of the environment under analysis. In other words, the speed of a user should be reduced if the neighborhoods two steps ahead are partially or fully occupied. Likewise, speed can increase when there is no occupation at least two steps forward.

Notice that panic situations are taken into account by the model. Indeed, as considered in here, the speed of each automaton may be as much as double of the speed in studies to simulate the natural walk. It is supposed that the speed used in here applies to non-typical situations in which people must run away. Notice also that all automata move without stopping until they will find a way out. They all have their only goal to exit the environment and interact to each other in such a way that those who do not know the way out follow the ones that know it. This situation applies to emergencies and not to usual behavior, when persons have their own objective inside the environment, following their own track. Besides, all individuals tend to approach the exit as much as they can at the same time forming congestion. They move around seeking the way out, even under congestion.

As a final note, it is clear that the ideas discussed here do not apply exclusively to pedestrian traffic applications. See, for instance, Schadschneider (2006) for an interesting application to vehicular traffic.

2.2. The Proposed Model

All components previously mentioned are part of a model that predicts the final movement probability, that is, the probability that a person moves to a position (x, y). The initial information is provided by the state matrix M of dimensions 3 x 3, which informs the initial probabilities, i.e., the prior probabilities of movement for a person in the environment to a position (x, y). An example of matrix M may be seen in Figure 2-a, in which the probabilities of moving from cell to cell as well as the likelihood of non-movement (i.e., the probability of maintaining the same position) are determined. The values in matrix M should be defined by the analyst based on all available information on the phenomenon under analysis.

The posterior probabilities are calculated using Schadschneider's (2002) model, which provides a way to update prior probabilities according to the following equation:

$$P_{xy} = N \exp\left(k_d D_{xy}\right) \exp\left(k_s S_{xy}\right) M_{xy} \left(1 - n_{xy}\right) \xi_{xy}. \tag{1}$$

0.03	0.07	0.10
0.05	0.15	0.40
0.03	0.07	0.10

a) matrix M_1 (3x3)

0.01	0.01	0.02	0.04	0.07
0.01	0.01	0.03	0.06	0.10
0.01	0.01	0.05	0.08	0.13
0.01	0.01	0.03	0.06	0.10
0.01	0.01	0.02	0.04	0.07

b) matrix M_2 (5x5)

Figure 2: *Examples of matrix M, which contains the prior probability of moving from one cell to another*

Note that, P_{xy} is the posterior probability that a person moves to position (x, y). M_{xy} represents the priori probability that a person moves to position (x, y), obtained from matrix M (see examples in Figure 2). D_{xy} is a numeric value that represents the strength of the dynamic field for position (x, y), which is obtained from a matrix D of equal size of the environment matrix. Finally, S_{xy} is a numeric value that represents the strength of the static field for position (x, y), which is also obtained from matrix D. Matrix S_{xy} is a numerical value (or weight) that is determined beforehand and remains fix. Generally, high weights are assigned to positions in the array environment that are close to the exits or located in more central areas. The closer to obstacles or walls is the position (x, y), the lower are the values of D_{xy}. A constant k_s represents the contribution of this weight. Note that if k_s equals zero, it means that the influence of the static force is eliminated from the posterior probability P_{xy}. The constant N is simply a normalizing constant that ensures that $\Sigma_{x,y}P_{xy} = 1$. The number of people in cell (x, y) is represented by n_{xy}, but because each cell only holds at most one person at a time, the possible values are only 0 or 1. Finally, ξ_{xy} is a factor that takes into account obstacles and barriers such that it is 0 if there is an obstacle at position (x, y) and 1 otherwise.

Note that there is an internal counter for each position (x, y) in the environment matrix that accumulates points each time a position is visited by a cell that is stored in the same position (x, y) of matrix D. The more times a position (x, y) is visited, the greater is the probability of it being visited again from another person. Thus, matrix D contains the weights for each position (x, y) of the environment matrix, which can change each step. It is possible to decrease or increase the contribution of the weight of D_{xy} by means of k_d Note that if k_d is equal to zero, we eliminate the influence of the dynamic force on the outcome of the model. Notice that for simulations in which the influence of a trail is more evident, as in a colony of insects, we can use high k_d values. However, for simulations of traffic flow, the trail should be strong so as to conveniently simulate environments in which the knowledge of persons of that environment is not as high (for example, shopping malls, fairs, and other events) as when people tend to follow the crowd in emergency events.

A rather straightforward assumption is that the trail effect decreases in most cases as time goes by. We assume that at some time, the effect of the trail will be reduced because people will be aware already about the best directions to follow, and as a result, the flow of movement will become almost independent of the trail effect. In Schadschneider's (2002) model, this effect is included by means of the following equation:

$$k_d^{(t)} = k_d^{(t-1)} \times Q, \tag{2}$$

where $k_d^{(t)}$ represents the value at the current step t, and $k_d^{(t\text{-}1)}$ represents the value at the previous step t-1 such that $P_{xy} \rightarrow M_{xy}$, when $t \rightarrow \infty$. Factor Q ($0 \leq Q \leq 1$) represents the decay rate associated with k_d.

Note that the usual 3x3 matrix M (Fig. 2-a) only provides the probabilities of moving to an immediate neighboring cell. However, if matrix M is extended to dimension 5x5 (Fig. 2-b), we can assign probabilities of moving to a neighborhood within a two-step distance of the current position in any direction. In this case, we derive a speed that is twice as high for such a person. The 3x3 matrix M is called M_1, and M_2 represents the 5x5 matrix M (see Fig. 2-b). Thus, a modified equation for the posterior probabilities is derived as:

$$P_{xy} = N \exp\left(k_d D_{xy}\right) \exp\left(k_s S_{xy}\right) M_{1_{xy}}^{V_{xy}} M_{2_{xy}}^{(1-V_{xy})} \left(1 - n_{xy}\right) \xi_{xy}, \qquad (3)$$

where M_{1xy} and M_{2xy} represent the numerical values of position (x, y) of the matrices M_1 and M_2, respectively. The term V_{xy} only assumes the values of 0 or 1 to indicate which of the matrixes M_1 and M_2 are used to compute P_{xy} for position (x, y). The term V_{xy} assumes a value of 0 if the entire neighborhood two steps away is unoccupied, whereas the term assumes a value of 1 if there is any occupation within two steps surrounding the position (x, y). Note that matrices M_1 and M_2 may not occur simultaneously.

Based on this procedure, the average speed of a person should double if there is available space to move (which is determined by matrix M_2). Otherwise, the mean speed decreases.

3. Results and Discussion

The two models Eq. (1) and Eq. (3) were encoded in C++; the software is available directly from the web[2] or from the authors upon request for educational and research purposes. In the simulations performed in this paper, we assign Q = 0.99 to represent a slow reduction of the trail effect. This value is usually arbitrary; further analysis of this issue should be undertaken in future studies. Note that matrix M (i.e., M_1 or M_2) must be rearranged during the simulation. The numerical values of the array are rearranged according to the position occupied by the cell. Each division of the space contains a numeric code ranging from 1 to 8 that determines the preferred direction of a cell given its position in time. This scheme is depicted in Figure 3. These codes are provided by the analyst before simulation based on a table in ASCII format. This process represents human vision in the artificial world, as it is through vision that we recognize the preferred directions of displacement. Fig. 3 shows the arrows indicating the preferred direction for

[2]URL: http://www.est.ufmg.br/ftp/duczmal/auto.

each code. Note that the values of probabilities in the matrix *M* turn as we turn the direction.

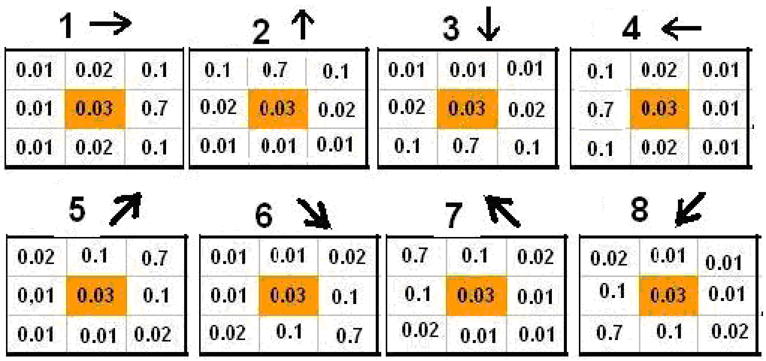
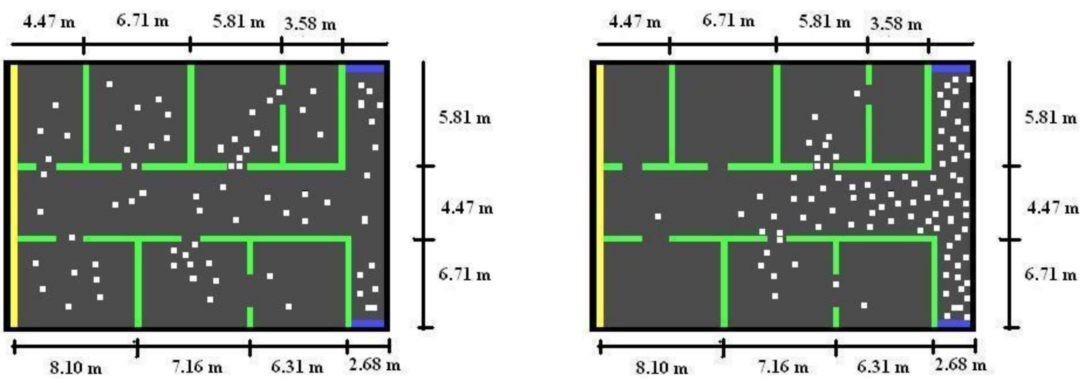


Figure 3: *Configuration of matrix M for each cell location*

The example we show here is the simulation of a rather complex environment presented in Fig. 4. For the sake of the argument, the example has only one floor. However, the methodology is applicable to multi-store environments, by accordingly considering the differences in the speed in ramps and stairs (which must be slower than in flat surfaces). In the figure, the doors are identified by traces of darkness. We remark that such an environment can be remodeled without requiring any changes in C++ code. Any configuration or reconfiguration of the environment may be performed by the user with the help of a program such as Microsoft® Excel. Fig. 4 shows snapshots at two crucial moments, i.e., right after the beginning of the simulation (Fig. 4-a) and approximately halfway through the simulation (Fig. 4-b).



a) at the beginning of simulation *b) approximately halfway through the simulation*

Figure 4: *Simulation shots at two different times*

To study overall evacuation time, we must establish some relationship between the time in minutes and number of steps of the simulation. We know that the movement of a person follows a discrete process in which displacement is rendered frame by frame. Each step of the movement represents a step in the simulation. According to Tregenza (1976), the average walking speed of a lone occupant in an environment may be assumed to be about 1.5 m/s. Thus, some person that covers the distance of $\sqrt{0.2}$ meters (that is, a simple shift on the 0.2 m^2 grid that discretizes the environment) would take about 0.298 seconds to complete, resulting in Eq. (4), which provides the total evacuation time. Variable n_{sim} represents the total number of discrete steps performed in the simulation before the latest cell leaves the environment.

$$\text{total evacuation time (min.)} = \frac{n_{sim} \times 0.298}{60}. \tag{4}$$

All simulations were carried out until all persons leave the environment, and then Eq. (4) was applied to provide the estimates of the total evacuation time in minutes. Table 1 shows the results of 1,000 replications for the models described by Eq. (1) and Eq. (3) with 5% and 10% occupation of the total space capacity. These densities were obtained from empirical studies (results not shown) regarding the maximum capacity of an environment in normal working conditions.

Table 1: *Mean evacuation time (min) as a function of the occupation, for models (1) and (3)*

Density		**Eq. (1)**	**Eq. (3)**	***p*-value (*t*-test)**
5%	Mean	1.3927 min.	0.7955 min.	0.0001
	StdDev	0.0731 min.	0.0536 min.	
10%	Mean	1.6401 min.	0.9721 min.	0.0002
	StdDev	0.0914 min.	0.0546 min.	

It is clearly shown in Table 1 that both models, i.e., the traditional model (Eq. (1)) and the new model (Eq. (3)), result in significantly different estimates (p-value < 0.05) for the mean evacuation time. This result reinforces the significant effect of adding a new component to the model. As such, there may be some bias in the estimation of evacuation time if one does not take into account the changes in the average walking speed persons may experience in crowded environments. We see that variability is lower in the case in which we use the model described in Eq. (3). This result can be explained by the fact that Eq. (3) assumes that people who are away from the crowd will have an almost entirely clear way ahead of them so that they develop a higher speed until they reach an overcrowded area and, consequently, develop a lower speed. This phenomenon is noticeable during real-time simulation.

Another aim of this study is to compare the variable obtained in the simulations, that is, evacuation time (or the elapsed time until the latest person vacates the environment), with a particular probability distribution function. The idea is to use some extreme value distribution, which is used to describe the distribution of maximum values in many practical situations, such as height of floods, temperatures, the age of oldest person to die in a city, and so on (Tomazella & Achcar, 1996.). This distribution would be most appropriate for a study about evacuation times. An acceptable fit to some extreme value distribution for the data obtained from the simulation model would be an indication that the simulation model may represent the real-life phenomenon. A random variable that follows the distribution of the maximum extreme values has the following probability density function:

$$f(x) = \frac{1}{\theta}\exp\left(\frac{-(x-\varepsilon)}{\theta}\right)\exp\left(-\exp\left(\frac{-(x-\varepsilon)}{\theta}\right)\right), -\infty < x < \infty, \qquad (4)$$

where $\theta\,(\theta > 0)$ is a scale parameter, and $\varepsilon\,(-\infty < \varepsilon < \infty)$ is a location parameter.

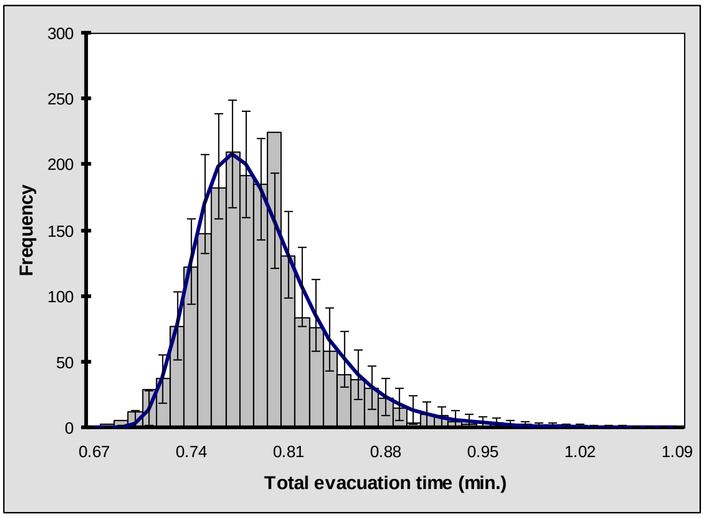


Figure 5: *Simulated total evacuation time (min.), fitted curve, and error bars*

A histogram of the total evacuation time, the fitted curve according to the extreme value distribution, and the error bars are shown in Fig. 5, which is based on a 2,000 step simulation performed under the same conditions used to obtain the results in Table 1 (results not shown are similar for other configurations). The model appears to show good fit. In fact, a χ^2 goodness-of-fit test shows that the model is suitable to describe the set of data obtained in the simulations (for details on χ^2 goodness-of-fit tests and an interesting application in fraud detection, see Geyer & Williamson, 2004).

Mean-square errors (SQE) were computed for the extreme value distribution and some other common probability density functions. The results can be seen in Table 2. Notice that from an analysis of SQE, the extreme value distribution produced the lowest value among all distributions tested. This value is an indication that the extreme value distribution may be one of the most appropriate models for adjusting evacuation times estimated from a simulation of the model described by Eq. (3).

Table 2: *Mean-square errors for simulated data on evacuation time and four probability density functions*

Distribution	Mean Square Error
Gama	379.0
Lognormal	273.1
Weibull	656.5
Extreme Value	183.9

4. Conclusions and Final Remarks

This paper discusses improvements in a multi-grid model for pedestrian dynamics based on bionics-inspired cellular automata. The newly-added feature is useful to model congestion effects, which include a well-known reduction in the average pedestrian walking speed and an increase in the number of people in an environment (for recent studies that also explore congestion, see van Woensel & Cruz, 2009, Cruz et al., 2010ab). A program was encoded and made available for research and educational purposes. Computational results have shown that the proposal is quite promising. After analyzing the data generated by the simulation model, we conclude that the inclusion of the new features seems to contribute significantly to making the evacuation simulations more realistic. In fact, adjusting the model to an extreme value distribution indicates a close similarity between the simulation results and the theoretical probability model.

The program developed here is useful for future studies, such as the investigation of the effect of environmental settings at the time of evacuation. Indeed, the effect of the population density in conjunction with environmental settings is known to be very important, as the evacuation time may be strongly affected by the location of exits and obstacles. Additionally, it should be worthwhile to analyze methods to search for ideal evacuation routes in order to optimize evacuation times. The topics mentioned above are examples of interesting topics for future research in this area.

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References

Cruz, F. R. B., Oliveira, P. C., Duczmal, L., 2010a. State-dependent Stochastic Mobility Model in Mobile Communication Networks. Simulation Modelling Practice & Theory 18(3), 348-365.

Cruz, F. R. B., Smith, J. M., Medeiros, R., 2005. An M/G/c/c state dependent queueing network simulation model. Computers & Operations Research 32(4), 919-941.

Cruz, F. R. B., van Woensel, T., Smith, J. M., Lieckens, K., 2010b. On the System Optimum of Traffic Assignment in *M/G/c/c* State-dependent Queueing Networks, European Journal of Operational Research 201(1), 183-193.

Frank, G., Dorso, C., 2009. Statistical behavior of pedestrians under panic. In: 2009 Joint Statistical Meetings – JSM 2009, 3878-3883.

Geyer, C. L., Williamson, P. P., 2004. Detecting fraud in data sets using Benfords law. Communications in Statistics - Simulation and Computation 33 (1), 229-246.

Guo, R-Y., Tang, T-Q., 2012. A simulation model for pedestrian flow through walkways with corners. Simulation Modelling Practice and Theory 21(1), 103-113.

Hamagami, T., Hirata, H., 2003. Method of crowd simulation by using multiagent on cellular automata. IEEE/WIC International Conference on Intelligent Agent Technology (IAT’03) 46-52.

Hughes, R. L., 2002. A continuum theory for the flow of pedestrians. Transportation Research Part B: Methodological 36(6), 507-535.

Jiang Y. Q., Zhang P., Wong S. C., Liu R. X., 2010. A higher-order macroscopic model for pedestrian flows. Physica A 389(21), 4623-4635.

Kirchner, A., Klüpfel, H., Nishinari, K., Schadschneider, A., Schreckenberg, A., 2003. Simulation of competitive egress behavior: Comparison with aircraft evacuation data. Physica A 324, 689-697.

Kirchner, A., Schadschneider, A., 2002a. Cellular automaton simulations of pedestrian dynamics and evacuation processes. In: Fukui, M., Sugiyama, Y., Schreckenberg, M., Wolf, D. E. (Eds.) Traffic and Granular Flow ’01, 531-536.

Kirchner, A., Schadschneider, A., 2002b. Simulation of evacuation processes using a bionics-inspired cellular automaton model for pedestrian dynamics. Physica A 312, 260-276.

Li, X., Kuang, H., Fan, Y., 2012. Lattice hydrodynamic model of pedestrian flow considering the asymmetric effect. Communications in Nonlinear Science and Numerical Simulation 17(3), 1258-1263.

Rinaldi, P. R., Dalponte, D. D., Vénere, M. J. Clausse, A., 2007. Cellular automata algorithm for simulation of surface flows in large plains, Simulation Modelling Practice and Theory 15(3), 315-327.

Schadschneider, A., 2002. Bionics-inspired cellular automaton model for pedestrian dynamics. In: Fukui, M., Sugiyama, Y., Schreckenberg, M., Wolf, D. E. (Eds.) Traffic and Granular Flow ’01, 499-509.

Schadschneider, A., 2006. Cellular automata models of highway traffic. Physica A 372, 142-150.

Smith, A., James, C., Jones, R., Langston, P., Lester E., Drury, J., 2009. Modelling contra-flow in crowd dynamics DEM simulation. Safety Science 47, 395-404.

Song, W., Xu, X., Wang, B-H, Ni, S., 2006. Simulation of evacuation processes using a multi-grid model for pedestrian dynamics. Physica A 363(2), 492-500

Stepanov, A., Smith, J. M., 2009. Multi-objective evacuation routing in transportation networks. European Journal of Operational Research 198, 435-446.

Tian, H. H., He, H. D., Wei, Y. F., Yu, X., Lu, W. Z., 2009. Lattice hydrodynamic model with bidirectional pedestrian flow. Physica A: Statistical Mechanics and its Applications 388(14), 2895-2902.

Tomazella, V. L. D. ; Achcar, J. A., 1996. Extreme value regression models: An useful reparametrization for the survival function. Journal of Applied Statistics 23(1), 59-68.

Tregenza, P. R., 1976. The design of interior circulation. Van Nostrand Reinhold Company, New York, NY.

van Woensel, T., Cruz, F. R. B., 2009. A Stochastic Approach to Traffic Congestion Costs. Computers & Operations Research 36(6), 1731-1739.

Wolfram, S., 1994. Cellular automata and complexity. Addison-Wesley publishing Company, New York, NY.

Zarboutis, N., Marmaras, N., 2007. Design of formative evacuation plans using agent-based simulation. Safety Science 45, 920-940.

Zheng X., Liu M., 2010. Forecasting model for pedestrian distribution under emergency evacuation. Reliability Engineering & System Safety 95(11), 1186-1192.