A Simple Procedure to Improve the Performance of Klein's 2-of-2 Run Rule in the Shewhart $\bar{X}$ Control Chart [*]

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May 01, 2023

Abstract

Supplementary run-rules have been used since the 1950s to improve the performance of the basic Shewhart $\bar{X}$ control chart. A procedure known as Klein's 2-of-2, a simple and efficient supplementary rule, signals that the process is out-of-control only when two sequential sample averages are observed above the upper control limit (UCL) or below the lower control limit (LCL). The use of this rule results in a substantial reduction of the average run length (ARL) in detecting small process shifts when compared to the traditional $\bar{X}$ control chart. However, it presents a poor performance of the out-of-control average run length for moderate to large process shifts (the ARL_1 converges to 2 for large shifts in the mean). In order to minimize this issue, some authors introduce additional control limits in order to delimit decision zones in relation to deciding whether the process is in control or out-of-control. Despite a better performance for moderate to large process shifts, the process complexity is also higher. In this paper we include a simple procedure (with no additional control limits) in relation to the 2-of-2 rule by examining if all values of each unit of the sample are beyond the control limits whenever its average is beyond the control limits. The new procedure has a similar performance in terms of ARL for small shifts in the mean and a better performance for moderate to large shifts (the ARL_1 now converges to 1) when compared to the standard 2-of-2 rule (where the ARL_1 converges to 2).

Keywords: Shewhart charts; run rules; ARL; Markov chain.