Local Bootstrap Approaches for Fractional Differential Parameter Estimation in ARFIMA Models

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Abstract

In this paper we investigate bootstrap techniques applied to the estimation of the fractional differential parameter in ARFIMA models, d. The novelty is the focus on the local bootstrap of the periodogram function. The approach is then applied to three different semiparametric estimators of d, known from the literature, based upon the periodogram function. By means of an extensive set of simulation experiments, the bias and mean square errors are quantified for each estimator and the efficacy of the local bootstrap is stated in terms of low bias, short confidence intervals, and low CPU times. Finally, a real data set is analyzed to demonstrate that the methodology may be quite effective in solving real problems.

Key words: Time series analysis, Fractionally integrated ARMA process, bootstrap.

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1 Introduction

The ARFIMA(p, d, q) processes belong to the wide class of long-memory models, for which the observations are not asymptotically independent (Reisen, 1993; Beran, 1994). Mandelbrot & Ness (1968) were two of the pioneers to present a model to adjust time series with long dependency. They have introduced the fractional Gaussian noise. In the 80's, Granger & Joyeux (1980) and Hosking (1981) showed that the ARFIMA$(0, d, 0)$ process presents the same behavior as the fractional Gaussian noise, while Geweke & Porter-Hudak (1983) proved the equivalence of these two stochastic processes.

A crucial open question that concerns the estimation of the fractional differential parameter, d, is how to construct confidence intervals or to perform hypothesis testing on the parameter. In this paper we will attach the problem by means of bootstrap approaches, which are resampling procedures (Efron, 1979) successfully applied over the past years in many areas, including time series in general (Franco & Souza, 2002) and ARFIMA models in particular (Franco & Reisen, 2004).

Thus, the main contribution of this paper is to present extensive computational experiments that show evidence in favor of the bootstrap methods developed by Souza & Neto (1996) and Paparoditis & Politis (1999), and here applied for the first time to the estimation of d in long-memory time series. Additionally, bootstrap confidence intervals for d are examined and an application to a real data set is discussed in details.

The paper is outlined as follows. In Section 2, long-memory models are described along with conveniently selected methods for parameter estimation. The bootstrap methods are detailed in Section 3. Section 4 is dedicated to present simulation evidences of the efficacy of the local bootstrap method. Section 5 presents an application to a real data set. Section 6 concludes the paper with final remarks and topics for future research in the area.

2 Long-memory Models and Parameter Estimation

2.1 ARFIMA(p, d, q) *Models*

In accordance to Beran (1994), long-memory phenomenon was known before stochastic models were even developed. Researchers in several fields had noticed that the correlation between observations sometimes decayed at a slower rate than for data following classical ARMA models. Later on, as a direct result of the pioneer research of Mandelbrot & Ness (1968), self-similar and long-memory processes were introduced in the field of statistics as a basis for inferences. Since then, this field is experiencing a considerable growth in the number of research results (for instance, see Franco & Reisen, 2004; Reisen et al., 2006, and many references therein).

As a natural extension of Box & Jenkins (1976) ARIMA models, let $\{X_t\}$ be the ARFIMA(p, d, q) process defined by

$$\phi_p(B)(1 - B)^d X_t = \theta_q(B)\varepsilon_t, \ d \in (-0.5, 0.5), \tag{1}$$

in which $\{\varepsilon_t\}$ is a white noise process normally distributed with zero mean and finite variance σ_ε^2. Respectively, $\phi_p(B)$ and $\theta_q(B)$ are the autoregressive and moving average polynomials of order p and q, B is the back-shift operator defined by $B^j X_t = X_{t-j}$, and $(1 - B)^d$ is the fractional differential operator.

In ARFIMA(p, d, q) models, d may assume fractional values and when $d \in (0.0, 0.5)$ they are known as long-memory models. In such cases, the process defined in (1) is stationary and invertible, with spectral density given by

$$f(\omega) = f_U(\omega)\Big[2\sin(\omega/2)\Big]^{-2d}, \ \omega \in (-\pi, \pi), \tag{2}$$

in which $f_U(\omega)$ is the spectral density function of ARMA(p, q) process, and $U_t = (1 - B)^d X_t$. For an in-depth discussion about ARFIMA models, the reader is encouraged to check Hosking (1981) and Reisen (1994). For a recent book with a review of different approaches, see Doukhan et al. (2003).

2.2 Estimation of the Differential Parameter

There have been proposed in the literature many estimators for the fractional differential parameter. We shall concentrate in estimators based upon the estimation of the spectral density function (2), convenient for the bootstrap approaches investigated here, as it will be seen shortly.

2.2.1 Geweke & Porter-Hudak's Method - GPH

The first estimator examined here, called GPH, was proposed by Geweke & Porter-Hudak (1983). Their method consists of taking the logarithm of the spectral density function (2), and estimating d by means of the regression equation obtained. Thus, the logarithm of $f(\omega)$ is

$$\ln f(\omega) = \ln f_U(0) - d \ln \Big[\sin(\omega/2) \Big]^2 + \ln \left[\frac{f_U(\omega)}{f_U(0)} \right]. \quad (3)$$

and because $f(\omega)$ is unknown, Geweke & Porter-Hudak (1983) proposed to estimate it by the periodogram function

$$I(\omega) = \frac{1}{2\pi} \left| \sum_{k=1}^{n-1} X_t e^{-iwf} \right|, \quad (4)$$

which gives

$$I(\omega) = \frac{1}{2\pi} \left[R(0) + 2 \sum_{k=1}^{n-1} R(k) \cos(kw) \right], \quad (5)$$

in which $R(.)$ denotes the sample autocovariance of X_t and n is the sample size. The GPH estimator is obtained by the regression equation between $\ln I(\omega)$ and $\ln \Big[2 \sin(\omega/2) \Big]^2$.

2.2.2 Reisen's Method - SPR

The second estimator considered, SPR, was proposed originally by Reisen (1994) and is based upon the smoothed periodogram function

$$f_{\text{sp}}(\omega) = \frac{1}{2\pi} \sum_{k=1}^{n-1} \lambda(k) R(k) \cos(kw), \quad (6)$$

in which $\lambda(k)$ is given by the Parzen lag window

$$\lambda(k) = \begin{cases} 1 - 6\left(\frac{k}{m}\right)^2 + 6\left(\frac{|k|}{m}\right)^3, & \text{if } |k| \leq m/2, \\ 2\left(1 - \left(\frac{|k|}{m}\right)\right)^3, & \text{if } m/2 \leq |k| \leq m, \\ 0, & \text{otherwise}, \end{cases}$$

$m = n^\beta$, $0 < \beta < 1$. The SPR estimator is obtained by the regression equation between $\ln f_{\text{sp}}(\omega)$ and $\ln \Big[2 \sin(\omega/2) \Big]^2$.

2.2.3 *Lobato & Robinson's Method - LBR*

The last estimator that will be considered here, called LBR, was proposed by Robinson (1994) and Lobato & Robinson (1996). This estimator is the weighted averages of the unlogged periodogram based upon the number of frequencies, the bandwidth τ, and a constant $q \in (0.0, 1.0)$

$$\mathrm{LBR}(q) = 0.5 - \frac{1}{2\ln q} \ln\left[\frac{\hat{F}(q\omega_\tau)}{\hat{F}(\omega_\tau)}\right], \tag{7}$$

in which $\hat{F}(\omega_\tau) = \frac{2\pi}{n}\sum_{j=1}^{[\omega_\tau/2\pi]} I(\omega_j)$ and $[.]$ means the integer part. Usual choices are $\tau = n^\alpha$ and $q = 0.5$ (Lobato & Robinson, 1996).

3 Bootstrap Methods

Bootstrap methods are resampling techniques, proposed originally by Efron (1979), designed to approximate the probability distribution function of the data by an empirical function of a finite sample. Their use in time series must be judicious because the observations are not independent and the time series structure may be lost in a careless resampling. Thus, the time series must be resampled indirectly. Among the promising research results on bootstrapping related to time series, we could mention Souza & Neto (1996), Paparoditis & Politis (1999), and Franco & Reisen (2004). Following, we will describe bootstrap techniques for ARFIMA models, for an in-depth simulation study.

3.1 *Nonparametric Bootstrap in the Residuals*

In order to avoid resampling directly from the time series, one possibility is to perform the resample from the residuals of the adjusted model. Thus, let $\{X_t\}$ be a time series with n observations modeled as an ARFIMA(p, d, q) model, as defined in (1).

After properly estimating the parameters ϕ_p, θ_q, and d, the residuals are easily estimated from

$$\hat{\varepsilon}_t = \hat{\theta}_q^{-1}(B)\hat{\phi}_p(B)(1-B)^{\hat{d}}X_t. \tag{8}$$

We then resample $\hat{\varepsilon}_t$ with replacement and construct from the resamples ε_t^* the bootstrap time series

$$X_t^* = \hat{\theta}_q(B)\hat{\phi}_p^{-1}(B)(1-B)^{-\hat{d}}\varepsilon_t^*. \quad (9)$$

Because no distribution was specified for the residuals, $\hat{\varepsilon}_t$, the approach is called non-parametric.

3.2 Parametric Bootstrap in the Residuals

Similarly, a parametric version of the bootstrap may be derived, as follows. The residuals are modeled as $\varepsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2_{\varepsilon_t})$. Likewise, after estimating the parameters ϕ_p, θ_q, and d, the residuals may be calculated by (8), from which the variance may be estimated, $\hat{\sigma}^2_{\tilde{\varepsilon}_t}$. We then resample with replacement from distribution $\mathcal{N}(0, \hat{\sigma}^2_{\tilde{\varepsilon}_t})$ and finally from (9) the bootstrap time series may be constructed.

3.3 Local Bootstrap in the Sample Spectral Density Functions

Yet another way of bootstrapping, proposed by Paparoditis & Politis (1999), is based upon the smoothed periodogram function, $f_{\text{sp}}(\omega)$, defined in (6), and on the asymptotic independence of its ordinates. 'Local' is due to the way that the resampling is done, as explained below. The method will be described only for $I(\omega_j)$ but it is equivalent for $f_{\text{sp}}(\omega)$.

Let $I(\omega_j)$, $j = 1, \ldots, N$, be the periodogram ordinates of $\{X_t\}$, in which $N = [n/2]$ and $[.]$ means the integer part. Assuming that the spectral density function, $f(\omega)$, defined in (2) is smooth, the periodogram replicates can be obtained locally, i.e., by sampling the frequencies that are in a neighborhood of the frequency of interest, ω.

Thus, the local bootstrap can be summarized as follows. The procedure is also illustrated in Figure 1.

(1) Select a resampling width k_n, in which $k_n \in \mathbb{N}$ and $k_n \leq [N/4]$.
(2) Define i.i.d. discrete random variables $S_1, \ldots, S_N$ that assume values in the set $\{0, \pm 1, \ldots, \pm k_n\}$.
(3) Each one of the $2k_n + 1$ ordinates can be resampled with equal probability

$$p_{k_n,s} = \frac{1}{2k_n + 1}.$$

(4) The bootstrap periodogram is defined by

$$I^*(\omega_j) = I(\omega_{j+S_j}),\ j = 1, \ldots, [n/2],$$
$$I^*(\omega_j) = I(-\omega_j),\ \omega_j < 0,$$
$$I^*(\omega_j) = 0,\ \omega_j = 0.$$

Paparoditis & Politis (1999) have showed that the local bootstrap is asymptotically valid and that some care should be taken for the choice of the resampling widths k_n, in the case of a finite sample size n. In particular, an optimal resampling width can be obtained from

$$k_{n,j} = n^{4/5} \left\{ \frac{9 f^2(\omega_j)}{8\pi^4 \left[f^{(2)}(\omega_j) \right]^2} \right\}^{1/5}, \tag{10}$$

in which it is assumed that $f^{(2)}(\omega_j) \neq 0$.

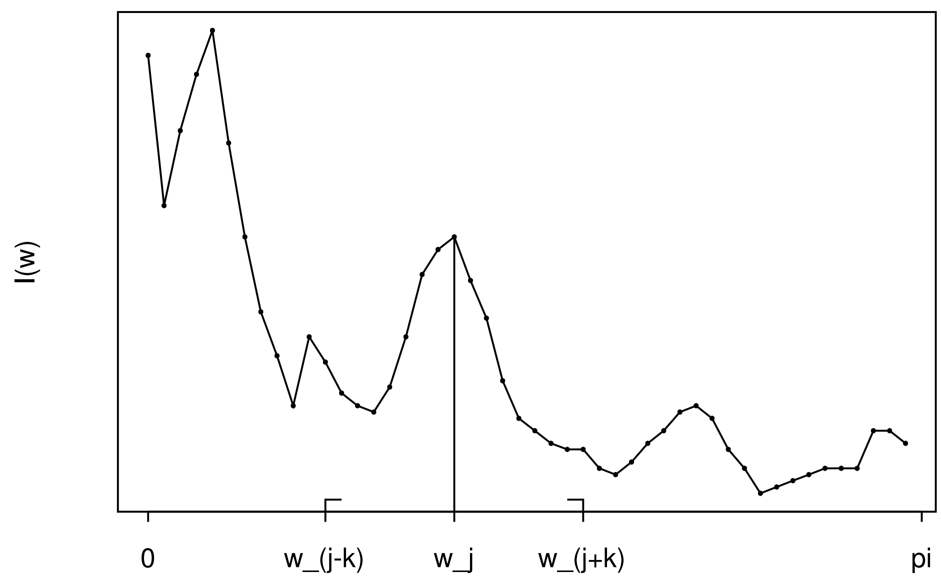

Fig. 1. Local bootstrap in the periodogram function.

3.4 Bootstrap Confidence Intervals

Not only are the punctual estimates needed in practice but also needed are the precision measures. Thus one may want to compute confidence intervals, in a definition taken from Bickel & Doksum (1977):

Definition 1 *Let $T(X)$ be an estimate of a parametric function $q(\psi)$. The random interval $[\underline{T}, \overline{T}]$ composed by the pair of statistics $\underline{T}$ and $\overline{T}$, with $\underline{T} \leq \overline{T}$ is a confidence interval of level $(1-\alpha)100\%$ for $q(\psi)$ if for all ψ, $P_\psi[\underline{T} \leq q(\psi) \leq \overline{T}] \geq 1-\alpha$.*

In this paper, the confidence intervals will be built based upon the bootstrap. As defined by Efron & Tibshirani (1993), for each estimator of d, we will generate Q independent bootstrap samples $X^{*1}, X^{*2}, \ldots, X^{*Q}$, and will estimate the fractional differential parameter for each one of them, $\hat{d}^{*i}$, $i = 1, 2, \ldots, Q$. The lower and upper bounds of the percentile bootstrap confidence intervals will be given by $[\hat{d}^{*(\alpha/2)};\ \hat{d}^{*(1-\alpha/2)}]$, in which $\hat{d}^{*(\beta)}$ is the $Q.(\beta)$-th ordered value of the bootstrap replications $\hat{d}^{*i}$.

4 Simulation Evidence

In order to attest for the efficiency of the bootstrap methods detailed in Section 3, we conducted experiments with simulated data. In the simulation study, we generated 1000 different ARFIMA$(0, d, 0)$ time series, by the algorithms of Hosking (1984) and Reisen (1993), with parameter $d = 0.2$, and sizes $n = 300$ and $n = 500$. The number of bootstrap replications was $Q = 1000$ (Efron & Tibshirani, 1993). The fractional differential parameter was then estimated by the estimators described in Section 2, GPH, SPR, and LBR. The statistics used to compare the bootstrap methods were the bias $[E(\hat{d}) - d]$ and the mean square error (MSE). FORTRAN was the programming language employed being the code available from the authors upon request.

To select the best resampling widths in the local bootstrap, experiments were conducted for the estimators GPH, SPR, and LBR, and $k = 1$, $k = 2$, $k = 5$, $k = 15$, $k = 40$, and k_j, with $j = 1, \ldots, [n/2]$. The results are presented in Figure 2. Firstly, notice that the bias are always negative and that by reducing the resampling width k the bias is reduced and the variance is increased. These results are in accordance with Paparoditis & Politis (1999). Additionally, in a direct comparison with Monte Carlo simulations (MC), it appears that $k = 1$ and $k = 2$ are the best resampling widths. From now on, we shall use only these two widths for local bootstrapping.

Once selected the best resampling widths, we shall compare all bootstrap methods. In Table 1, we see the bias and MSE for all cases tested. The Monte Carlo estimates are in accordance with known results (see, e.g., Smith et al., 1997; Franco & Reisen, 2004). Initially, we notice from Table 1 that, in pairs, the bootstrap methods in the residuals (nonparametric and parametric) and

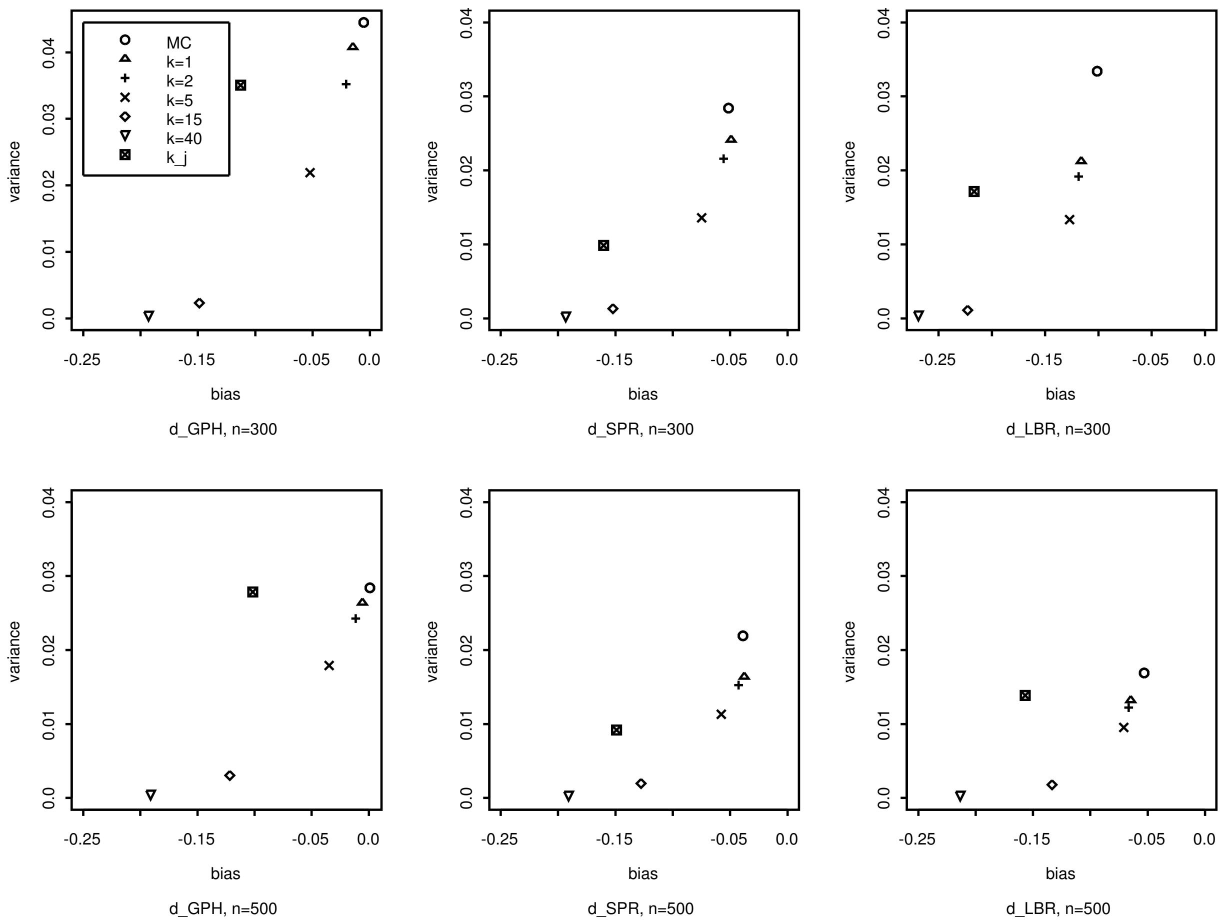

Fig. 2. Estimator performances for the local bootstrap compared with Monte Carlo (MC) simulations.

Table 1
Bias and MSE for $\hat{d}$.

			Estimator		
n	Method		GPH	SPR	LBR
300	Monte	Bias	-0.0054	-0.0514	-0.1011
	Carlo	MSE	0.0445	0.0284	0.0334
	Nonparametric	Bias	**-0.0138***	-0.1071	-0.1824
	Bootstrap	MSE	0.0350	**0.0295***	0.0485
	Parametric	Bias	-0.0139	-0.1071	-0.1822
	Bootstrap	MSE	0.0351	**0.0295***	0.0484
	Local	Bias	-0.0148	**-0.0492***	**-0.1161***
	Bootstrap, $k = 1$	MSE	**0.0410***	0.0265	0.0347
	Local	Bias	-0.0207	-0.0554	-0.1187
	Bootstrap, $k = 2$	MSE	0.0356	0.0247	**0.0333***
500	Monte	Bias	0.0010	-0.0389	-0.0531
	Carlo	MSE	0.0284	0.0219	0.0169
	Nonparametric	Bias	-0.0539	-0.1242	-0.1189
	Bootstrap	MSE	0.0189	**0.0237***	0.0199
	Parametric	Bias	-0.0536	-0.1243	-0.1189
	Bootstrap	MSE	0.0189	0.0238	0.0199
	Local	Bias	**-0.0056***	**-0.0377***	**-0.0647***
	Bootstrap, $k = 1$	MSE	**0.0264***	0.0178	0.0174
	Local	Bias	-0.0116	-0.0426	-0.0666
	Bootstrap, $k = 2$	MSE	0.0244	0.0171	**0.0166***

*Closest values to Monte Carlo.

the local bootstrap methods ($k = 1$ and $k = 2$) provided similar results. For $n = 300$, the local bootstrap methods (for both k) produced the best bias values comparing to Monte Carlo simulations, for the estimators SPR and LBR, but for the estimator GPH, the nonparametric bootstrap in the residuals had the best performance. For the MSE the best performances were observed for the local bootstrap methods, for the estimators GPH and LBR.

By increasing the sample sizes to $n = 500$, the superiority of the local bootstrap is more pronounced. The local bootstrap method with $k = 1$ simply presented the best bias values comparing to Monte Carlo simulations, for all estimators (see in Table 1 values in bold).

As a note on the computational efficiency of the bootstrap methods considered, additional advantages for the local bootstrap methods are their low and well-behaved CPU times. With the help of an IMSL-FORTRAN procedure, we estimated that the average CPU time for the local bootstrap was $\approx 0.59\%$ of the time spent by the bootstrap in the residuals. In other words, the average speed for the bootstrap in the residuals was ≈ 170 times higher than the average speed for the local bootstrap. Additionally, we noticed that the CPU time increased slower with the time series sizes n for the local bootstraps than for the bootstrap in the residuals. For instance, we noticed that the average CPU time increased by 62% for the bootstrap in the residuals, while the increase was 28% for the local bootstrap, as the time series sizes increased from $n = 300$ to $n = 500$.

5 Empirical Studies

The time series under study is presented in Figure 3, which is composed by 288 wind speed measurements, in meters per second, for each five minutes, from 00:00:00 h to 23:55:00 h, in May/17/1991, by the SILSOE Research Institute. These data can be found in the work of Reisen (1993), which presents a selection of adjustments of ARFIMA(p, d, q) models and shows that one of the best models is the ARFIMA$(1.0, d, 1.0)$. The estimate for the fractional differential parameter around $\hat{d} = 0.3$ indicates that the time series presents long-memory behavior. Forecasts for the data set may be found in Reisen & Lopes (1999).

In Table 2, we can see the bootstrap 95% confidence intervals for d by the nonparametric, parametric, and local bootstrap methods. It appears that the local bootstrap method with $k = 1$ produces the shortest confidence intervals. From Figure 4 it is seen that the estimates are quite assymmetric around the punctual estimates for all bootstrap methods.

Also noticeable from Table 2 is that different estimators provided different evidences about the significance of d. For all bootstrap confidence intervals obtained from the LBR estimator, the parameter did not seem to be significant. However, from the simulation study presented in Figure 2, the LBR estimates are not the most reliable as they produced the largest bias. Also unreliable is the confidence interval from the GPH and nonparametric bootstrap, which presented the largest width.

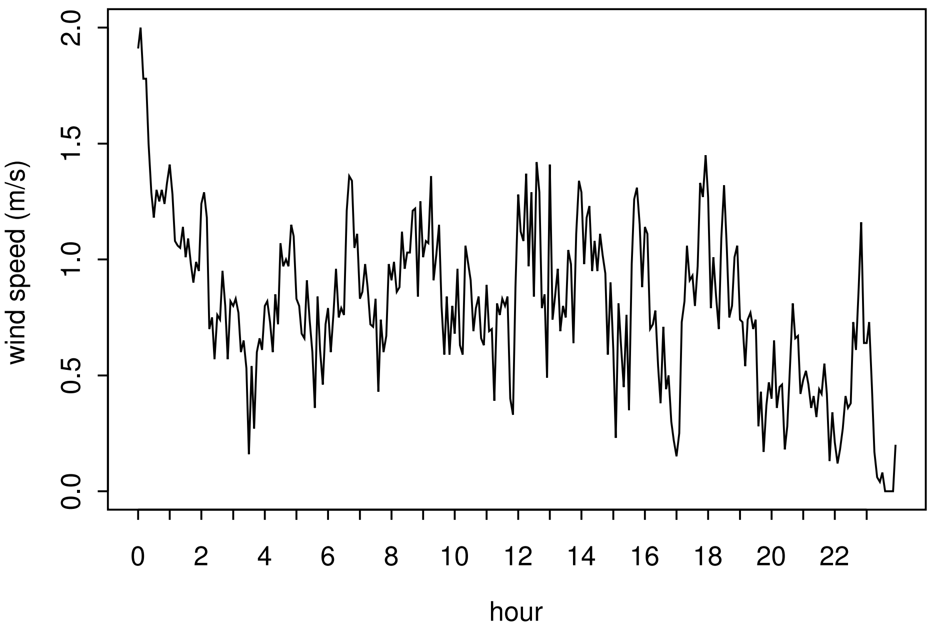

Fig. 3. Wind speed data.

Table 2
Bootstrap 95% confidence intervals and widths for d.

Bootstrap Method		Estimates		
		GPH	SPR	LBR
		0.2886	0.2990	0.1751
Nonparametric	[95% ci]	[-0.0296; 0.7270]	[0.1083; 0.5485]	[-0.1963; 0.3470]
	width	0.7566	0.4402	0.5433
Parametric	[95% ci]	[0.0169; 0.7189]	[0.1135; 0.5517]	[-0.1847; 0.3442]
	width	0.7020	0.4383	0.5289
Local with $k = 1$	[95% ci]	[0.1116; 0.4270]	[0.2195; 0.3576]	[-0.0279; 0.2695]
	width	**0.3154***	**0.1381***	**0.2974***
Local with $k = 2$	[95% ci]	[0.0805; 0.4622]	[0.1703; 0.3860]	[-0.0575; 0.2748]
	width	0.3817	0.2156	0.3323

*Shortest confidence intervals.

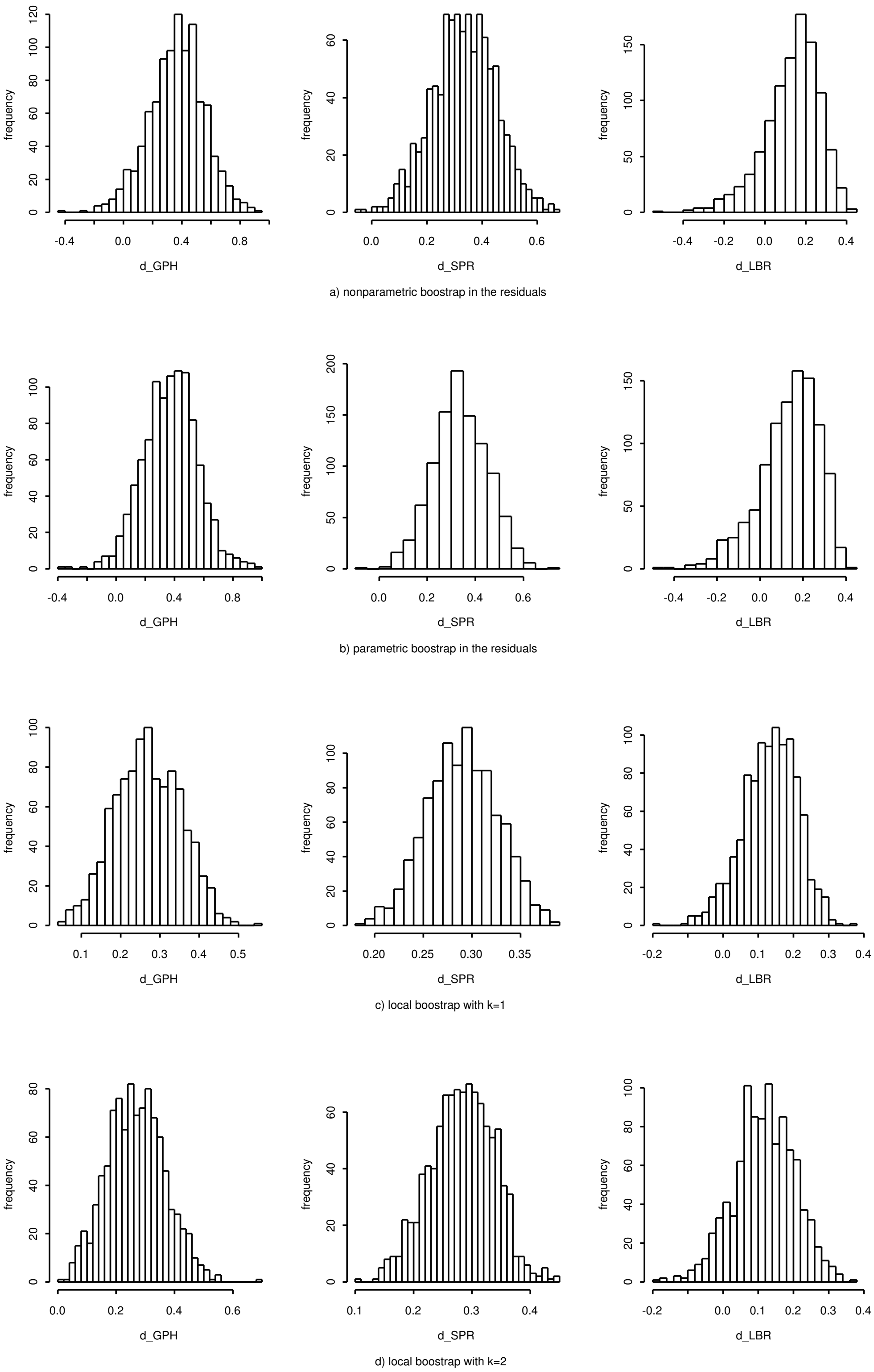

Fig. 4. Bootstrap estimates for d.

6 Conclusions and Final Remarks

The main goal of this paper was to find efficient bootstrap approaches for inferences on the fractional differential parameter in ARFIMA(p, d, q) models, d. The bootstrap in the residuals has been used before in a similar context but from the best knowledge of the authors it is the first time that the local bootstrap in the periodogram function has been applied for this class of model. In the evaluation of different resampling widths, k, when k is small or the size of the series is large, it was seen that the estimates provided are the best compared to Monte Carlo simulations. Comparing the performance of the bootstrap in the residuals with the performance of the local bootstrap methods, we assessed the superiority of the latter not only in terms of precision of the estimates but also in terms of computational efficiency. Another disadvantage of the bootstrap in the residuals is that it is dependent on the model. That is, poorly adjusted models will lead to poor bootstrap estimates for d. In the application to real time series, the local bootstrap methods also were superior, presenting the shortest confidence intervals and the lowest CPU times. Topics for future research in the area include the use of the local bootstrap method under different estimators for d, as there are many of them based upon the periodogram function, extensions to the seasonal fractionally integrated processes (Reisen et al., 2006), and the use of maximum likelihood methods (Doornik & Ooms, 2003), as they are quite different from the methods examined here and they have convenient asymptotic properties.

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