Full Predictivistic Modeling of Stock Market Data: Application to Change Point Problems

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Abstract

In change point problems in general we should answer three questions: how many changes are there? Where are they? And, what is the distribution of the data within the blocks? In this paper, we develop a new full predictivistic approach for modeling observations within the same block of observation and consider the product partition model (PPM) for treating the change point problem. The PPM brings more flexibility into the change point problem because it considers the number of changes and the instants when the changes occurred as random variables. A full predictivistic characterization of the model can provide a more tractable way to elicit the prior distribution of the parameters of interest, once prior opinions will be required only about observable quantities. We also present an application to the problem of identifying multiple change points in the mean and variance of a stock market return time series.

Key words: Uncertainty modeling, normal-inverse-gamma distribution, product partition model, student-t distribution.

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1 Introduction

The student-t distribution is a class of model that can be obtained as a location and scale mixture of the normal distribution in which the mixing measure

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is the normal-inverse-gamma distribution. Consequently, the student-t distribution can be constructed in two stages. Firstly, given the location and scale parameters, a conditional normal distribution is considered. Secondly, a prior distribution for these location and scale parameters is specified.

In a predictivistic characterization of the process, the first stage is replaced by an invariance or sufficiency assumption over an infinite sequence of potentially observable random quantities. For the student-t model above mentioned the first stage is replaced by the orthogonal invariance, which preserves the vectors of ones (see, for example, the works of Smith, 1981, and Diaconis et al., 1992, among others). However, this invariance condition is not enough to identify the mixing measure. Thus some extra conditions on observable random variables are necessary to obtain the mixing measure (see the papers by Diaconis & Ylvisaker, 1979, and Arellano-Valle et al., 1994, for instance). In a full predictivistic approach, Loschi et al. (2003b) stated the conditions to characterize such a student-t distribution.

This paper basically revisits previous contributions (Loschi et al., 2003a,b, 2006) used as a basis for an original predictivist justification to the successful methods developed in the past for multiple change-point identification, but without any formal justification such as the one present here. Additionally, the paper shows how this apparently hard way of modeling could be applied to model the behavior of stock market returns and other similar data sequences. The multiple change-point problem is treated here in the light of the well-known product partition model (PPM), proposed by Barry & Hartigan (1993). However, it is worthwhile noting that some other models also deal with the change point problem and, in some aspects, do it in a much more natural way, the so-called Dynamic Linear Model, which can be seen in the book by West & Harrison (1999), along with other classes of dynamic models in Bayesian time series analysis and forecasting.

Some of the advantages of the PPM are that it allows the identification of multiple change points in the parameters, as well as in the functional form of the distribution function itself. Besides, the PPM brings flexibility into the analysis because the number of change points is a random variable, unlike threshold models (Chen & Lee, 1995) and others (Hawkins, 2001; Zmeškal, 2005) that consider the number of change points fixed. Firstly, Barry & Hartigan (1993) applied the PPM to the identification of multiple change points in the means of normal random variables with common variances. Afterwards, Crowley (1997) provided a new implementation of a Gibbs sampling scheme for the PPM in order to estimate the normal means, which was extended by Loschi et al. (2003a) to estimate both the means and the variances. Extensions of the PPM to a more general context can also be found in the papers by Quintana & Iglesias (2003) and Loschi et al. (2006).

In order to illustrate the methodology, we apply the predictivistic model developed to identify multiple change points both in the means μ and variances σ^2 of normal data sequentially observed. A conjugate prior distribution is considered for the parameters, μ and σ^2, which is justified within a full predictivistic setting. In fact, a more tractable way to elicit the prior distribution of μ and σ^2 is proposed, once opinions are required only about observable quantities. Yao's algorithm (Yao, 1984) is used to compute the posterior estimates and a Gibbs sampling scheme is applied to estimate the posterior distributions of the number of change points and the instants when the changes occurred. The algorithms were implemented in C++ and the code is available from the authors upon request. The method was applied to identify multiple change points in the means and variances of a series of returns of the Chilean stock market. Different prior distributions were considered for the probability that a change occurs in any instant of the time and a sensitivity analysis was provided. As a result, it was seen that the returns in the Chilean stock market are characterized by changes in the expected returns (means) and volatilities (measured here as variances).

The paper is organized as follows. In Section 2, the PPM is briefly reviewed and a predictivistic characterization of the student-t PPM is provided, which explains in an alternative way the choices adopted for the prior distributions. In Section 3, the methodology is illustrated to the identification of change points in the mean return and volatility of ENDESA returns (the Chilean National Electricity Company). The return behavior within each block is modeled taking into account the predictivistic approach. A sensitivity analysis to the PPM is also provided. Finally, Section 4 closes the paper with final concluding remarks.

2 The Student-t PPM

In this section we apply the PPM to identify multiple change points in the mean and variance of normal data observed sequentially through time. We consider a conjugate analysis and present a new full predictivistic characterization to the complete model (the likelihood function and prior distribution).

2.1 The product partition model (PPM)

Let $X_1, \ldots, X_n$ be a data sequence. Consider a random partition ρ of the set $I = \{1, \ldots, n\}$ and a random variable B that represents the number of blocks in ρ. Consider that each partition $\rho = \{i_0, i_1, \ldots, i_b\}$, $0 = i_0 < i_1 < \cdots < i_b = n$, divides the sequence $X_1, \ldots, X_n$ into $B = b$, $b \in I$, contiguous subsequences, which will be denoted by $\mathbf{X}_{[i_{(r-1)}i_r]} = (X_{i_{(r-1)}+1}, \ldots, X_{i_r})'$, $r = 1, \ldots, b$. Let $c_{[ij]}$ be the prior cohesion associated to the block $[ij] = \{i+1, \ldots, j\}$, $i, j \in I \cup \{0\}$, $j > i$, which represents the degree of similarity among the observations within $\mathbf{X}_{[ij]}$ and can be interpreted as the transition probabilities in the Markov chain generated by the change points.

Hence, it is said that the random quantity $(X_1, \ldots, X_n; \rho)$ follows a PPM, denoted by $(X_1, \ldots, X_n; \rho) \sim PPM$, if:

i) the prior distribution of ρ is the following product distribution

$$P(\rho = \{i_0, \ldots, i_b\}) = \frac{\Pi_{j=1}^{b} c_{[i_{(j-1)}i_j]}}{\sum_{\mathcal{C}} \Pi_{j=1}^{b} c_{[i_{(j-1)}i_j]}}, \tag{1}$$

in which $\mathcal{C}$ is the set of all possible partitions of I into b contiguous blocks with end points $i_1, \ldots, i_b$ that satisfy the condition $0 = i_0 < i_1 < \cdots < i_b = n$, $b \in I$;

ii) conditional on $\rho = \{i_0, \ldots, i_b\}$, the sequence $X_1, \ldots, X_n$ has joint density given by

$$f(X_1, \ldots, X_n | \rho = \{i_0, \ldots, i_b\}) = \Pi_{j=1}^{b} f_{[i_{j-1}i_j]}(\mathbf{X}_{[i_{j-1}i_j]}), \tag{2}$$

in which $f_{[ij]}(\mathbf{X}_{[ij]})$ is the joint density of the random vector $\mathbf{X}_{[ij]} = (X_{i+1}, \ldots, X_j)'$.

Note that the number of blocks in ρ, B, has prior distribution given by

$$P(B = b) \propto \sum_{\mathcal{C}_1} \Pi_{j=1}^{b} c_{[i_{(j-1)}i_j]}, \; b \in I, \tag{3}$$

in which $\mathcal{C}_1$ is the set of all partitions of I into b contiguous blocks.

The posterior distributions of ρ and B have the same form of the prior distribution, in which the posterior cohesion for block $[ij]$ is given by $c^*_{[ij]} = c_{[ij]} f_{[ij]}(\mathbf{X}_{[ij]})$. That is, as observed by Barry & Hartigan (1993), the PPM induces conjugacy.

In the parametric approach of the PPM, a sequence of unknown parameters $\theta_1, \ldots, \theta_n$ is considered and the sequence of random variables $X_1, \ldots, X_n$ has conditional marginal densities $f_1(X_1|\theta_1), \ldots, f_n(X_n|\theta_n)$. In this case, two observations X_i and X_j, such that $i \neq j$, are considered in the same block if they are believed identically distributed. Thus, the block predictive distribution, Eq. (2), can be obtained as follows

$$f_{[ij]}(\mathbf{X}_{[ij]}) = \int_{\Theta_{[ij]}} f_{[ij]}(\mathbf{X}_{[ij]}|\theta) \; \pi_{[ij]}(\theta) d\theta, \tag{4}$$

in which $\Theta_{[ij]}$ is the parameter space corresponding to the common parameter, say, $\theta_{[ij]} = \theta_{i+1} = \cdots = \theta_j$, which indexes the conditional density of $\mathbf{X}_{[ij]}$.

The prior distribution of $\theta_1, \ldots, \theta_n$ is constructed as follows. Given a partition $\rho = \{i_0, \ldots, i_b\}$, $b \in I$, we have that $\theta_i = \theta_{[i_{(r-1)} i_r]}$ for every $i_{(r-1)} < i \leq i_r$, $r = 1, \ldots, b$, and that $\theta_{[i_0 i_1]}, \ldots, \theta_{[i_{(b-1)} i_b]}$ are independent, with $\theta_{[ij]}$ having (block) prior density $\pi_{[ij]}(\theta)$, $\theta \in \Theta_{[ij]}$.

Hence, the goal is to obtain the marginal posterior distributions of the parameters ρ, B, and θ_k, $k = 1, \ldots, n$. The product estimates of θ_k are given by

$$E(\theta_k | X_1, \ldots, X_n) = \sum_{i=0}^{k-1} \sum_{j=k}^{n} r^*_{[ij]} \; E(\theta_k | \mathbf{X}_{[ij]}), \tag{5}$$

in which $r^*_{[ij]} = P([ij] \in \rho | X_1, \ldots, X_n)$ is known as the posterior relevancies. In order to compute $r^*_{[ij]}$, we will use the recursive algorithm developed by Yao (1984) but an alternative scheme to compute the posterior relevancies based on Gibbs sampling was given by Loschi et al. (2003a).

2.2 A Predictivistic Justification for the Student-t PPM

Sometimes it is not an easy task eliciting the prior distributions to solve real problems. In this section we establish a new full predictivistic characterization of the student-t PPM for which the likelihood function and prior distribution of $\theta = (\mu, \sigma^2)$ are consequences of judgments on observable quantities. As a by-product this characterization provides a tractable way to elicit the prior distribution of θ.

As mentioned in the previous section, the student-t distribution is a location and scale mixture of normal distributions for which the mixing measure is the normal-inverse-gamma distribution. Thus, it follows that the student-t distribution can be obtained in two stages. First, given the location and scale parameters, a conditional normal distribution is specified. Second, we identify a normal-inverse-gamma distribution as the prior joint distribution for the location and scale parameters. By adopting the predictivistic approach, the first stage is replaced by an assumption about observables. For example, the assumption of invariance under some groups of orthogonal transformation over infinite sequences of random quantities implies that the law of sequence of observables can be represented as mixtures of conditionally normally distributed and independent quantities (for instance, see Smith, 1981).

However, this type of condition does not allow the characterization of the mixing measure. Additional conditions must be assumed to obtain the mixing measure. Based on Diaconis & Ylvisaker (1979), Arellano-Valle et al. (1994) characterized a scale mixture of a normal distribution by considering invariance under the orthogonal transformation and some additional conditions to predict X_{n+1}^2. In the full predictivistic approach of Arellano-Valle et al. (1994) the mixing measure (prior distribution) obtained was the inverse-gamma distribution. Arellano-Valle et al. (1994) also obtained a characterization for a location and scale mixture of normal distributions. However, because the characterization depends on non-observable quantities it is not a full predictivistic characterization of the model. Proposition 1 below improves such partial result.

Consider $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$ and $S_n^2 = \sum_{i=1}^n (X_i - \bar{X}_n)^2$. We say that an infinite sequence of random variables, $X_1, X_2, \ldots$, is $\mathcal{O}(\mathbf{1})$-invariant if for each $n \geq 2$ and real values m and r, the conditional distribution of $\mathbf{X}_{[0n]}$, given $\bar{X}_n = m$ and $S_n^2 = r^2$, is uniform on the n-sphere centered in $m\mathbf{1}_n$ and with ratio r, that is, on the set $\mathcal{S}_n = \{(x_1, \ldots, x_n) \in \mathbb{R}^n : \bar{x}_n = m, \sum_{i=1}^n (x_i - \bar{x}_n)^2 = r^2\}$.

Proposition 1 *Let $X_1, X_2, \ldots$ be an infinite sequence of $\mathcal{O}(\mathbf{1})$-invariant random variables, such that $P(X_1 = X_2) = 0$ and*

$$\begin{cases} E(X_3^2|X_1, X_2) = e(X_1^2 + X_2^2) + w, \\ E(X_3|X_1, X_2) = e(X_1 + X_2) + u, \end{cases} \tag{6}$$

then $e \in (0, 1/2)$, $u \in \mathbb{R}$, $w > u^2/(1-2e)$ and, for each $n \geq 3$,

$$\mathbf{X}_{[0n]} \sim t_n\left(\frac{u}{1-2e}\mathbf{1}_n;\ \mathbf{I}_n + \frac{e}{1-2e}\mathbf{1}_n\mathbf{1}_n';\ \frac{1}{e}(w - \frac{u^2}{1-2e});\ \frac{1+e}{e}\right). \tag{7}$$

The converse also holds.

Proof: *From Smith's Theorem (Smith, 1981), there are random variables μ and σ^2, such that, for every $n \geq 2$,*

$$\mathbf{X}_{[0n]}|\mu,\ \sigma^2 \sim \mathcal{N}(\mu\mathbf{1}_n,\ \sigma^2\mathbf{I}_n),$$

in which $\sigma^2 > 0$ with probability one. Consequently, considering that $M = \sum_{i=1}^2 X_i = 2\bar{X}$ and $Q = \sum_{i=1}^2 X_i^2 = S^2 + 2\bar{X}^2$ and denoting by $\theta = (\theta_1, \theta_2) = (\mu/\sigma^2, -1/2\sigma^2)$ the natural parameter of the distribution of (M, Q), given (μ, σ^2), we obtain the following conditional density of (M, Q) given θ:

$$dP_\theta(M,Q) = \exp\{(\theta_1,\theta_2)(M,Q)^t - D(\theta)\}\, d\xi(M,Q),$$

in which $d\xi(M,Q) = \frac{1}{\pi\sqrt{2}}(Q - M^2/2)^{-\frac{1}{2}}d\lambda$, λ *is the Lebesgue measure defined on* $\mathbb{R}^2$ *and* $D(\theta) = -\theta_1^2/(2\theta_2) - \log(-\theta_2)$.

The vector of partial derivates of $D(\theta)$ *with respect to the natural parameters* θ_1 *and* θ_2 *is given by*

$$D'(\theta) = \left(-\frac{\theta_1}{\theta_2}, \frac{\theta_1^2}{2\theta_2^2} - \frac{1}{\theta_2}\right) = E\{(M,Q)|\theta\}.$$

Hence, by using the properties of the conditional expectation and the conditions in Eq. (6), it follows that

$$\begin{aligned} E\{D'(\theta)|(M,Q)\} &= E\{E\{(M,Q)|\theta_1,\theta_2\}|(M,Q)\} \\ &= 2E\{E\{(X_3,X_3^2)|(\mu,\sigma^2)\}|X_1,X_2\} \\ &= 2e(X_2+X_1;\ X_2^2+X_1^2) + 2(u,w). \end{aligned}$$

From Theorem 3 in Diaconis & Ylvisaker (1979) the following prior density for (μ,σ^2) *is obtained*

$$\pi(\mu,\sigma^2) = \mathcal{K}\left\{\frac{1}{\sigma^2}\right\}^{\frac{1}{2e}+\frac{3}{2}} \exp\left\{-\frac{1}{2e\sigma^2}(w - \frac{u^2}{1-2e})\right\}\left\{\frac{1-2e}{e\sigma^2}\right\}^{\frac{1}{2}}$$
$$\exp\left\{-\frac{1-2e}{2e\sigma^2}\left(\mu - \frac{u}{1-2e}\right)^2\right\}.$$

Consequently, Eq. (7) is obtained. The converse is obtained by using the properties of the student-t distribution. ■

Proposition 1 extends some partial results from Arellano-Valle et al. (1994) by providing a full predictivistic characterization of a location and scale mixture of normal distributions. Extensions of this result to student-t linear models can be found in Loschi et al. (2003b). As a consequence of Proposition 1 the parameters μ and σ^2 have the normal-inverse-gamma distribution $\mu|\sigma^2 \sim \mathcal{N}\left(\frac{u}{1-2e}, \frac{e\sigma^2}{1-2e}\right)$ and $\sigma^2 \sim \mathcal{IG}\left(\frac{1}{2e}(w - \frac{u^2}{1-2e}), \frac{1+e}{2e}\right)$. Note that under $\mathcal{O}(\mathbf{1})$-invariance assumptions the representations in Eq. (6) are equivalent to such a specification.

Note also that if we assume that the sequence of observations is $\mathcal{O}(\mathbf{1})$-invariant and that the conditions in Eq. (6) are also a reasonable assumption for the sequence, the block predictive distribution considered in Eq. (4) is a student-t

distribution and the block posterior distribution indicated in Eq. (5), $f(\theta_k|\mathbf{X}_{[ij]})$, is a normal-inverse-gamma distribution.

3 Applications: The Chilean Stock Market Behavior

The algorithms were coded in C++ and all tests were carried out on a PC, 166 MHz, 32 MB RAM, running Windows 98, and using a freely available compiler (http://www.delorie.com/djgpp). In the Gibbs sampling scheme, we generate 5,000 samples starting from a vector of zeros. The convergence was reached after 1,000 iterations, which were discarded. A lag of 1 was selected since the correlation among the vectors was low. We have tried different starting points and confirmed in practice that once convergence is reached, different starting points will not lead to significantly different results (results not shown).

The ultimate goal of this section is to present a sensitivity analysis for the PPM, developed here for several different degenerate prior distributions for p, and to identify multiple change-points in the mean and variance (that is, the expected return and volatility, respectively) of the returns of the ENDESA stock series from 1987 to 1994, as seen in Figure 1. As usual in finance, the returns were defined by using the transformation $X_t = (P_t - P_{t-1})/P_{t-1}$, in which P_t is the price at month t. Defined in such a way, the returns within each block can be considered normally distributed, given the expected return and volatility.

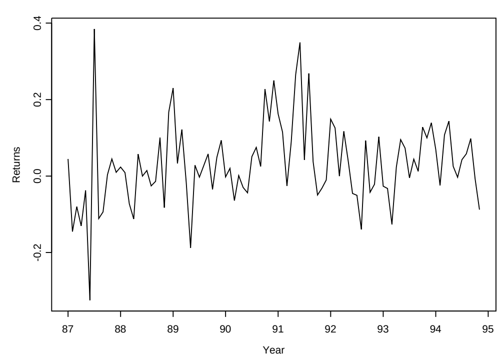

Fig. 1. Returns of ENDESA.

3.1 Change point and sensitivity analyses

In order to describe the uncertainty of the parameter, $\theta_{[ij]} = (\mu_{[ij]}, \sigma^2_{[ij]})$, we have adopted the same normal-inverse-gamma prior specification of Loschi and Cruz (2002), $\mu_{[ij]}|\, \sigma^2_{[ij]} \sim \mathcal{N}(0, \sigma^2_{[ij]})$ and $\sigma^2_{[ij]} \sim \mathcal{IG}(0.01/2, 4/2)$. A zero

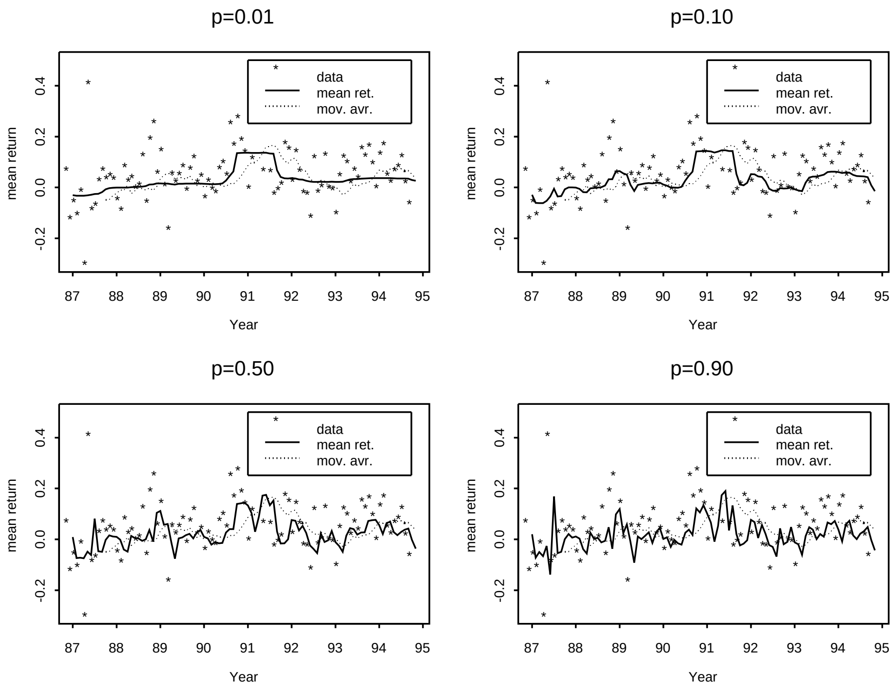

Fig. 2. Posterior means of μ.

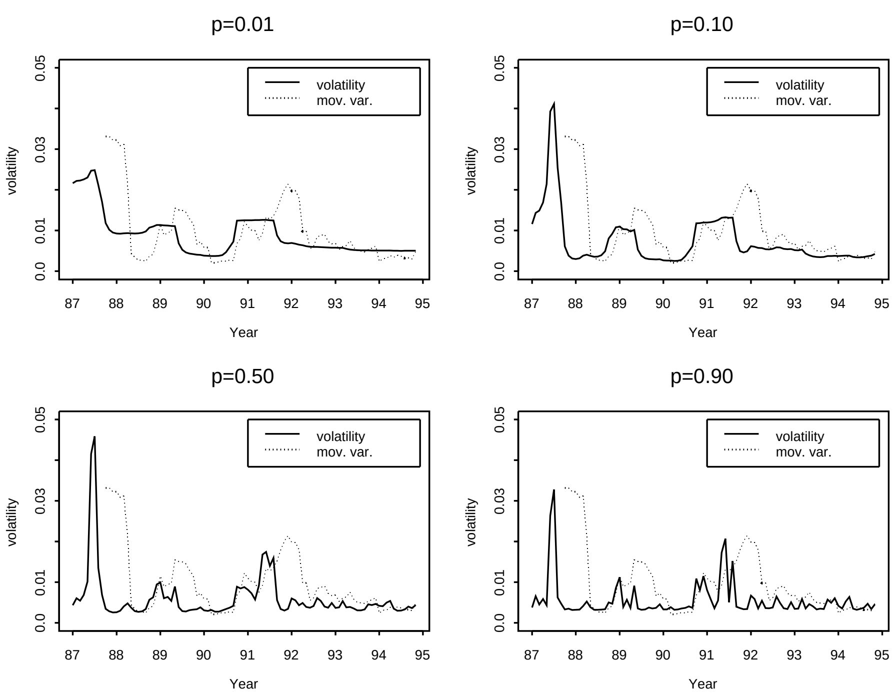

Fig. 3. Posterior means of σ^2.

mean prior was found reasonable for the data under analysis but the model supports different values for the μ. Different prior specifications here would

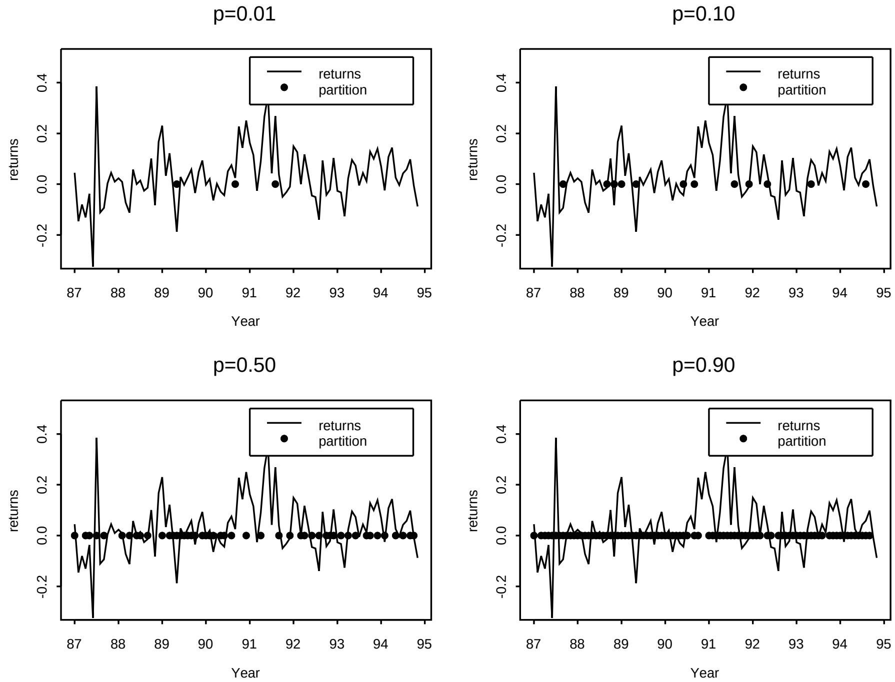

Fig. 4. Posterior distribution of ρ.

Table 1
Prior and posterior probability of the most probable partition.

	probability	
p	prior	posterior
0.01	4.007×10^{-7}	0.3567
0.10	1.593×10^{-16}	0.0173
0.50	2.524×10^{-29}	0.0013
0.90	1.161×10^{-13}	0.0285

lead to substantially different results, as noticed by Loschi and Cruz (2002). Additionally, we have considered as prior cohesions a truncated geometric distribution with parameter p, which are known as Yao's cohesions (Yao, 1984). In order to evaluate the influence of several different prior specifications on the posterior estimates of μ, σ^2, B, and ρ, we have chosen $p = 0.01$, $p = 0.1$ (both indicating that a small number of changes is expected), $p = 0.5$, and $p = 0.9$ (that is, prior specifications that suggest a high number of change points in the prior evaluation).

Figures 2 and 3 show the posterior estimates of μ and σ^2. The product estimates of μ (σ^2) were contrasted with the centered arithmetic moving average (variance) of order 10 for the means (variances), respectively. It is noticeable that more instants were identified as change points when high values of p were considered. We also noticed that similar estimates were obtained for similar

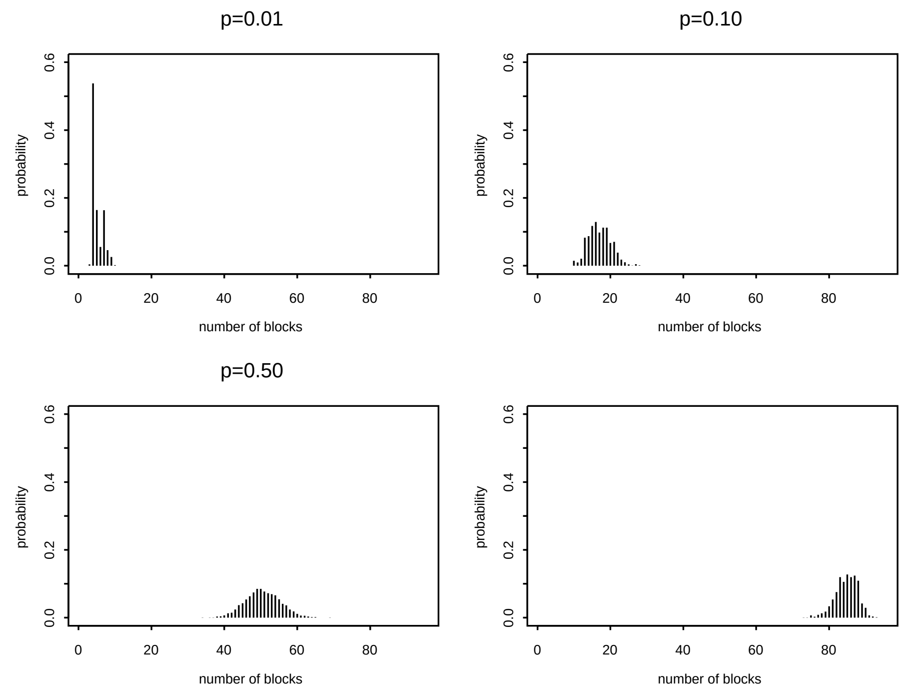

Fig. 5. Posterior distribution of B.

Table 2
Descriptive statistics for the posterior distributions of B.

	prior		posterior					
p	mean	variance	mean	variance	mode	median	Q1	Q3
0.01	0.94	0.931	5.093	2.158	4	4	4	6
0.10	9.40	8.460	17.08	9.682	16	17	15	19
0.50	47.0	23.50	50.52	23.44	50	50	47	54
0.90	84.6	8.460	84.75	9.263	85	85	83	87

values of p. For $p = 0.1$ we observed that the PPM estimates were very similar to the naïve centered arithmetic moving averages.

Figure 4 presents the most probable partition for different values of p. Similarly to the conclusions drawn earlier, note that high values of p led to more instants identified as change points. Table 1 presents the prior and posterior probabilities of the most probable partition. Note that the probability of occurrence of the most probable partition increased substantially from the prior to the posterior evaluation.

From Figure 5 we can note that the posterior distributions for the number of blocks, B (or for the number of change points, $B-1$) have only one mode, independently on the value assumed for p. We also can note that if p is small, the posterior distribution of B is centered in low values (see Table 2, for some descriptive statistics). It is also noticeable that for all values of p the probability of having one or more change points in the ENDESA series is 1.0. Note from Table 2 that the summaries of location (mean, mode, median) of the posterior distribution of B increase with p, as well as the mean of the prior distribution of B. We also observe that the posterior variance is higher than the prior variance for the cases $p = 0.01$, 0.10, and 0.90, but it is not true for $p = 0.50$. It is also noticeable that the posterior variance increases as p increases for all values of p but for $p = 0.90$. Finally, it is remarkable that the inference procedure is quite sensitive to the choice of p and when $p = 0.90$ we do not see much gain in information on the posterior mean of B. Note that not all the models are acceptable. In the example, when B is estimated as 85 for $p = 0.90$, it is quite hard to believe that almost all observations form different blocks. This might be seen as an indication that the model is inappropriate.

3.2 A note on the model specification

We suppose that, conditional on the average stock return and its total standard deviation, any path followed by the returns within a block presenting the same average returns and total standard deviation is equally likely to occur, which is mathematically expressed by the $\mathcal{O}(\mathbf{1})$-invariance assumption amongst the returns. Hence, assuming extendibility, that is, assuming that all subsequences $(X_{i+1}, \ldots, X_j)$ are part of an infinite $\mathcal{O}(\mathbf{1})$-invariant sequence, we have that the joint distribution of the ENDESA returns in the same block, $\mathbf{X}_{[ij]}$, can be represented as a mixture of the product of the normal distributions $\mathcal{N}(\mu_{[ij]}, \sigma^2_{[ij]})$ (Smith, 1981).

We also assume the conditions in Eq. (6) understanding that they elucidate the considerations by Mandelbrot (1963), as well as what was suggested by Maeda (1996) to be reasonable for the Chilean stock market, which is that large returns tend to be followed by large returns and small returns tend to be followed by small returns, and changes in this behavior are produced by unanticipated information. These assumptions lead to a predictive distribution with heavy tails (student-t distribution) for the returns in the same block which also discloses a structure of correlation amongst the returns. The Chilean stock market is emerging and so more susceptible to the political atmosphere. Consequently it can experience more changes than a developed market and the student-t distribution is more appropriate to describe the behavior of its stock returns (Duarte Jr. & Mendes, 1997; Mendes, 2000). Note that the normality assumption adopted by Hsu (1984) (see also Hawkins, 2001) to describe the behavior of the Dow Jones Industrial Average is stronger than our assumptions here that the data are conditionally normally distributed.

Yao's prior cohesions imply that the sequence of change points establishes a discrete renewal process with the occurrence times identically distributed following the geometric distribution. This type of product partition distribution is adequate to represent reasonably well the situation described by Mandelbrot (1963), and later by Maeda (1996) for the Chilean stock market, who established that the changes in the behavior of the series of stock returns are a consequence of information not previously anticipated, so that the past change points are non-informative about the future change points.

4 Conclusions

In this paper we have presented a new full predictivistic approach for the student-t model and showed how this way of modeling can be applied to model the behavior of the stock market returns. We applied this predictivistic approach for modeling the block of observations in change point problems. We identified multiple change points both in the means μ and variances σ^2 of normal data sequentially observed by using an extension of the product partition model (PPM) developed by Barry & Hartigan (1993). The methodology was applied to the identification of multiple change points in the mean returns and volatilities of the ENDESA stock returns. A sensitivity analysis was also provided.

We perceived that the predictivistic approach may provide a treatable way to construct the block predictive distribution as well as the block posterior distributions for the parameters of interest. We concluded that the prior specifications for p has a strong influence in the posterior distributions of both the number of change points and the instants when the changes occurred. Also heavily influenced are the product estimates of the means and variances.

Because p is crucial for the inferences, one could model p by means of a general prior distribution, which certainly would add a lot of flexibility to the model, as the analyst could also include uncertainly to the prior information. However, a predictivistic modeling for a model including prior distributions for p is still an open research question. In this case, the conjugacy of PPM model is lost and it is impossible to use Yao's procedure.

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