State-dependent Stochastic Mobility Model in Mobile Communication Networks

F. R. B. Cruz[*] [†] P. C. Oliveira[‡] L. Duczmal[‡]
fcruz@est.ufmg.br pcampos@ufmg.br duczmal@est.ufmg.br

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Abstract — **We apply finite state-dependent queueing networks to model mobility in mobile communication systems. Although they have been successfully used in the past to model vehicular traffic, state-dependent models have not been applied to mobile communication systems, to the best of our knowledge. The novelty of state-dependent stochastic mobility models is that the congestion phenomenon is explicitly considered, that is, the user speeds fall when the number of users in the system increases. We present a detailed description of the simulation model used to estimate the performance measures of the queueing networks and show computational results for a comprehensive set of instances. As we show, finite state-dependent stochastic models bring interesting new insights, for instance, that in some cases mixed bimodal distributions will better describe the cell residence time of a call than the classical probability distributions used in the past.**

Keywords — Simulation; performance; state dependent; queueing networks.

1 Introduction

Interest in studying the performance of cellular systems by means of stochastic models has increased significantly recently, with growing demand for quality services. Although the results are still modest and constrained to simple problem instances, our understanding of the area continues to increase (see for instance, Alfa and Liu [2]). It is a well-recognized fact that users' mobility has generated new challenges for the engineers in charge of designing, planning, dimensioning, and maintaining cellular networks. In mobile systems, users want to move around and still keep connected to the cellular system (see Fig. 1, adapted from Zonoozi and Dassanayake [28]). In order to meet this requirement, the cellular system must hold and periodically update information regarding all users. By means of mathematical models, one tries to predict the behavior of these users in order to reduce the amount of information collected and stored. In fact, it was recognized a decade ago that mobility models play one of the most important roles in the description and design of cellular systems [28]. Included among the parameters of interest in a cellular system, directly influenced by mobility, are the handover, offered traffic, dimensioning of signaling networks, user location updating, registration, paging, and multilayer network management [28].

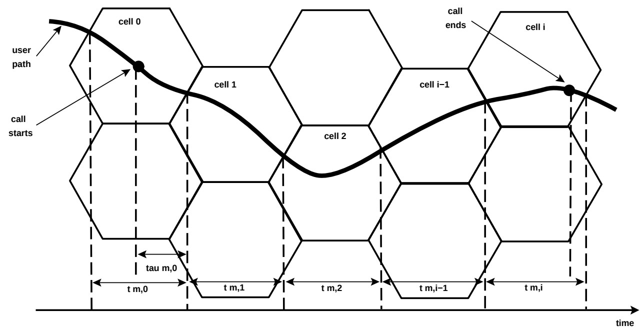

Figure 1: Mobility temporal diagram [28]

A review of the available literature in the area shows that many authors have dealt with mobility models for mobile communication networks, as presented in Fig. 2. Modern approaches usually consider stochastic models for the speed of the users. Both uniform and non-uniform probability distributions have been used. Exponentially and generally distributed speed models have also been developed. However, no research can be located that considers general state-dependent stochastic models with average speeds being a function of the number of users currently in the system. This is the model that we propose in this article.

The effects of user mobility on the handover performance were thoroughly investigated by Han Han [10]. However, the speed of the cars was considered to be exponentially distributed, and the speed of the pedestrians to be uniformly distributed. Although this may be acceptable as an approximation to make the model

[*]School of Computer Science and Information Technology, University of Nottingham, Jubilee Campus, Wollaton Road, Nottingham NG8 1BB, UK. On sabbatical leave from the Department of Statistics, Federal University of Minas Gerais, 31270-901 - Belo Horizonte - MG, Brazil.

[†]Corresponding author. E-mail: fcruz@est.ufmg.br. P: +55 31 3409 5929. F: +55 31 3409 5924.

[‡]Department of Statistics, Federal University of Minas Gerais, 31270-901 - Belo Horizonte - MG, Brazil.

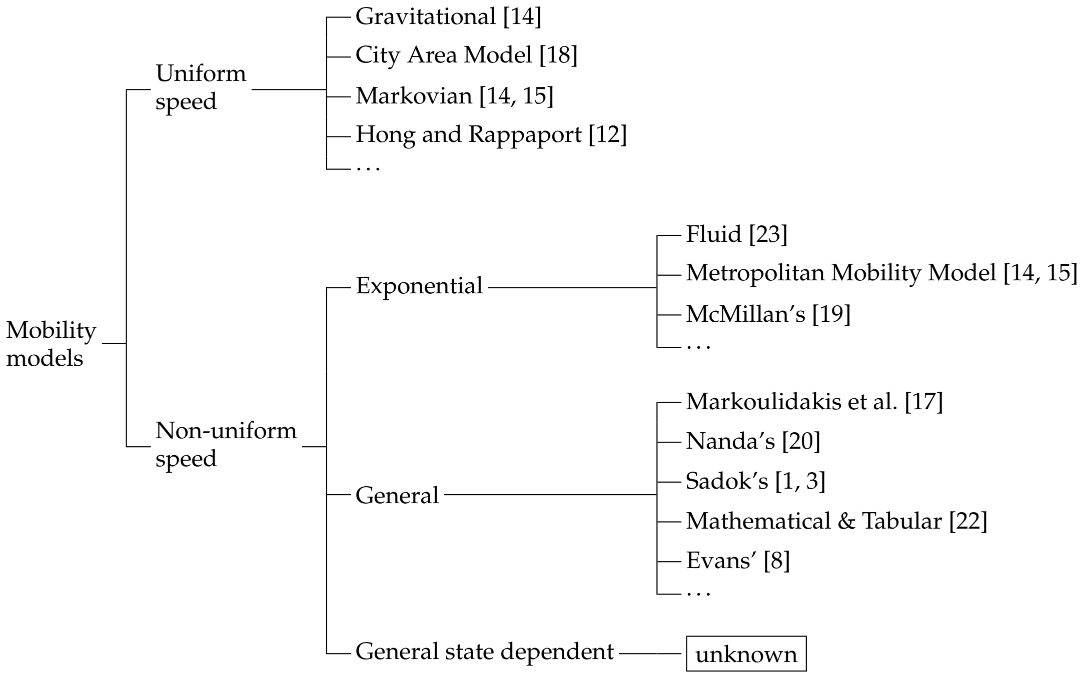

Figure 2: Some mobility models for cell residence time distribution

computationally tractable, it may lead to the conclusion that either the cell residence times or the channel holding times are Markovian, which is not true, according to results reported by Hegde and Sohraby [11]. In fact, simulation data analyzed by Zonoozi and Dassanayake [28] showed that the generalized gamma distribution is a better approximation for the cell residence time distribution and that the holding time of a cellular network is a negative exponential distribution.

However, as we show in the following sections, it does not seem that a single probability distribution would be appropriate to model a mobile communication network. The cell residence time is strongly affected by congestion effects within the cell. In other words, under very reasonable assumptions, the average speed of the cars should be considered state-dependent (see Fig. 3). We show that it is possible to end up with a hypoexponential distribution for the random variable *residence time* (that is, the random variable follows a probability distribution that has a lower variability than an exponential distribution), or even a bimodal probability distribution, as we simply vary the level of congestion of the system under analysis. Of course, the effect of these findings on the overall performance will not be small, given the strong link between users' mobility and quality of service in mobile networks [16].

This article is organized as follows. The state-dependent mobility simulation model is described in detail in Section 2. Section 3 presents experimental results obtained for small-scale systems of mobile communication. Section 4 gives the conclusion, which includes a highlight of the main results and a discussion of some open questions raised by the simulation study we conducted.

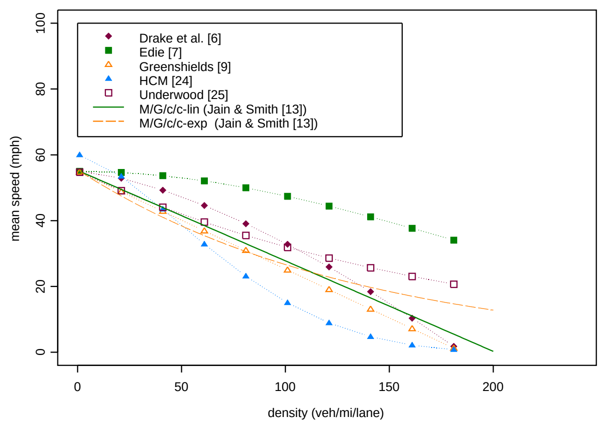

Figure 3: Empirical distributions for vehicular traffic flows [6, 7, 9, 24, 25] and $M/G/c/c$ state-dependent models [13]

2 A STATE-DEPENDENT MOBILITY MODEL

2.1 *Mobility parameters*

In order to perform an analysis of a mobile communication system, some parameters must be defined. In

Fig. 1, one sees a user traveling through a cellular network. Their path initiates on cell #0, in which the residence time is $T_{m,0}$, with the beginning of a new call. The channel holding time on this cell is given by $\tau_{m,0}$. As time goes by and the user moves throughout the route, the system will automatically switch his/her connection from cell #0 to cell #1, which is commonly called handoff (or handover). As users keep moving from cell to cell through the cellular network, they will eventually reach cell i, when the call is finalized. One of the most important parameters for describing user mobility is the random variable $T_{m,i}$, which represents the time that a user m spends in cell i. A major focus of this article is the development of a better and more accurate model for random variable $T_{m,i}$.

2.2 *Congestion models*

We shall use queueing networks to model traffic. For a recently published discussion upon the suitability and advantages of such models, the reader is referred to the article by van Woensel and Cruz [26]. A particular set of finite queues, known as $M/G/c/c$ state-dependent, has been used specifically to model congestion in vehicular traffic networks [13]. In Kendall notation, M is a Markovian arrival process, G is a general service time distribution, which here is state-dependent, c represents the number of parallel servers and, finally, c is also the total capacity of the systems, including those in service.

The most important feature of this model is that its mean speed decreases as the number of users in the system increases, as shown in Fig. 3 (adapted from Jain and Smith [13]), with empirical curves related to various North American roads. By using $M/G/c/c$ state-dependent queues, each road link may be seen as parallel servers for its occupants. The maximum number of parallel occupants equals the capacity of the road link, which is also the total amount of users allowed simultaneously in the system, and it is given by

$$c = \lfloor k \times l \times w \rfloor,$$

in which l is the length of the road, w is the number of lanes, c is the total capacity, and $\lfloor x \rfloor$ is the highest integer not larger than x. The constant k represents the jam density, which is the vehicular density at which flow comes to a halt. Different estimates of k have found values in the range of 115–165 vehicles/km-lane ($\approx$ 185-265 vehicles/mile-lane, as reported by Jain and Smith [13]; in this work we assume 200 veh/mile-lane). Note that the above discussion about capacity is only in terms of the physical number of cars that can fit on a given stretch of the road, which does not necessarily correspond to the 'call' capacity, the number of calls that a given base station can sustain at a given time.

In the congestion model, the traffic flows through the road link at an average speed V_n, which is a function of the number of vehicles n currently on the road and its capacity c. Based on empirical data, analytical linear and exponential models were developed [27] by means of the following notation:

$V_n \rightarrow$ average speed for an occupation of n vehicles;

$V_1 \rightarrow$ free flow average speed;

$V_a \rightarrow$ average speed for a density of a veh/km-lane;

$V_b \rightarrow$ average speed for a density of b veh/km-lane.

The values a and b are arbitrary points used to adjust the exponential curve. Both linear and exponential models usually will fit satisfactorily to empirical traffic data and produce fairly good results Jain and Smith [13]. For the sake of conciseness, we shall present here only the exponential model, the one actually used in this article:

$$V_n = V_1 \exp\left[-\left(\frac{n-1}{\beta}\right)^{\gamma}\right], \tag{1}$$

in which

$$\gamma = \ln\left[\frac{\ln(V_a/V_1)}{\ln(V_b/V_1)}\right] / \ln\left(\frac{a-1}{b-1}\right),$$

and

$$\beta = \frac{a-1}{[\ln(V_1/V_a)]^{1/\gamma}} = \frac{b-1}{[\ln(V_1/V_b)]^{1/\gamma}}.$$

In vehicular-related applications, commonly used values are $k = 200$ veh/mile-lane, as stated earlier, and $a = 20$ and $b = 140$, corresponding to vehicular densities of 20 veh/km-lane and 40 veh/km-lane, respectively. Looking at the curves presented in Fig. 3, reasonable values for such points are $V_a = 48$ mph and $V_b = 20$ mph.

The probability distribution of the number of users, as a function of λ, the arrival rate, is:

$$p(n) \equiv P[N=n] = \left[\frac{\Big(\lambda \times E[T_1]\Big)^n}{n!f(n)\dots f(1)}\right] \times p(0), \tag{2}$$

for $n = 1, 2, \dots, c$, in which

$$p(0) \equiv P[N=0] = 1/\left\{1 + \sum_{i=1}^{C}\left[\frac{\Big(\lambda \times E[T_1]\Big)^i}{i!f(i)\dots f(1)}\right]\right\}$$

is the empty system probability, λ is the arrival rate, $E[T_1]$ is the expected service time for a lone occupant in the system, and $f(n) = V_n/V_1$ is the service rate for n users simultaneously in the system.

By means of Eq. (2), it is possible to compute performance measures such as the blocking probability, throughput, expected number of users in the system (also known as work-in-process), and expected service time, among others. The blocking probability is the probability that an additional user will arrive when the number of users in the system is c, that is:

$$p_{\text{blocking}} \equiv p(c) \equiv P\left[N = c\right].$$

The throughput, also known as the effective arrival rate in the system, is given by:

$$\theta \equiv \lambda_{\text{eff}} \equiv \lambda[1 - p(c)].$$

The expected number of users in the system follows from the definition of expectation of a random variable:

$$L \equiv E[N] = \sum_{n=1}^{C} np(n).$$

The expected time in the system (that is, the expected service time) may be calculated from the definition of expectation, or simply from Little's Law:

$$W \equiv E[T] = \frac{L}{\theta}.$$

As a final remark, we remind the reader that the aim here is to computationally estimate these basic performance measures. It has not been assumed that all users on the road will have ongoing calls, which would be quite unrealistic. The relationship between the distribution of the number of customers in the system and the distribution of the number of users on the road with ongoing calls is complex and requires careful consideration. Further, in this formulation, handovers are not explicitly considered. These are certainly important issues, which will be dealt with in future work.

2.3 *Discrete-event simulation model*

We propose a simulation model, which extends the algorithm proposed by Cruz et al. [5], for $M/G/c/c$ state-dependent queueing networks. Essentially, the model implements the object `MgccSimul`, presented in Fig. 4. We now describe in detail the object `MgccSimul` and all the data structures involved, namely, `nOfNodes`, the number of $M/G/c/c$ state-dependent queues nodes, `totalTime`, the total simulation time, `arcs`, an origin-destination matrix, a vector of `nOfNodes` objects of type `MgccResource`, and, finally, `MgccEventQueue`, an event queue. Objects `MgccResource` keep track of all statistics of interest for each one of the queues, namely, `sumBloc`, the sum of blocking, `sumArr`, the number of arrivals, `sumDep`, the number of departures, `sumTime`, the accumulated amount of time in the system, and `users`, the current number of users in the system. Also part of each `MgccResource` object is `GenCM`, the congestion model, with methods to access c, the queue's capacity, $E[T_1]$, the expected service time for a lone occupant in the system, and V_n, the average speed (service rate) for the current number of users in the system.

The most critical part of object `MgccSimul` is object `MgccEventQueue`, which implements the event queue. The event queue has been implemented as a dynamic linked list, built in running time, with the task of book-keeping all the discrete events. Unexpectedly, after much experimentation, we found that it is considerably less time consuming to keep the event queue unsorted, at least for state-dependent queueing networks. Even considering the overhead of traversing the whole list to recover the next event to be processed, a worst-case $\mathcal{O}(n_{\texttt{list}})$ operation, it is more efficient to keep the list unsorted. This is because the list is shuffled every time an entity arrives into or leaves the system, since V_n, the service rates defined in Eq. (1), must be updated for each entity that remains in the system.

In the event queue, each object `MgccEvent` has the following variables: `whichQueue`, an indication of which $M/G/c/c$ queue it belongs to, `occurTime`, the expected time for the event to occur, `type`, the event type (among the possible events `arrival`, `departure`, and `end_simulation`), and `MgccEntity`, the entity to which it relates. Object `MgccEntity` represents each user (vehicle) in the $M/G/c/c$ state-dependent queueing network, which has the following variables: `id`, an unique numerical identification, `sisArrival`, the time when it arrived at the system, `queueArrival`, the time it arrived at the current queue, `lastChange`, the time when the last change occurred in the state (that is, when some entity joins or leaves that particular queue, there will be a change in its state), `lastPosition`, the physical position of the entity when the state in the queue last changed, and, finally, `celArrival`, the arrival time at the cell.

The cells are defined as a group of an arbitrary number of queues and this information is stored in matrix `Cells`. If a queue i belongs to cell j, then one has

$$\texttt{Cells}[\texttt{i}, \texttt{j}] = \texttt{TRUE}.$$

The algorithm is presented in pseudo-code in Fig. 5. Initially, the event queue `MgccEventQueue` is initialized with the last event (event type `end_simulation`) and the first events, which are the first arrivals (event type `arrival`). Then, iteratively, the earliest event is sought and processed until the final event (`end_-simulation`) is found. The procedures to deal with the arrivals, `ProcessArrival`, and departures, `ProcessDeparture`, will not be detailed here, for the sake of conciseness, as they are essentially no different from those described by Cruz et al. [5].

3 Computational experiments

We present here computational results from the experiments run with the discrete simulation model proposed. All algorithms were coded in C++ and are available from the authors upon request. All experiments were

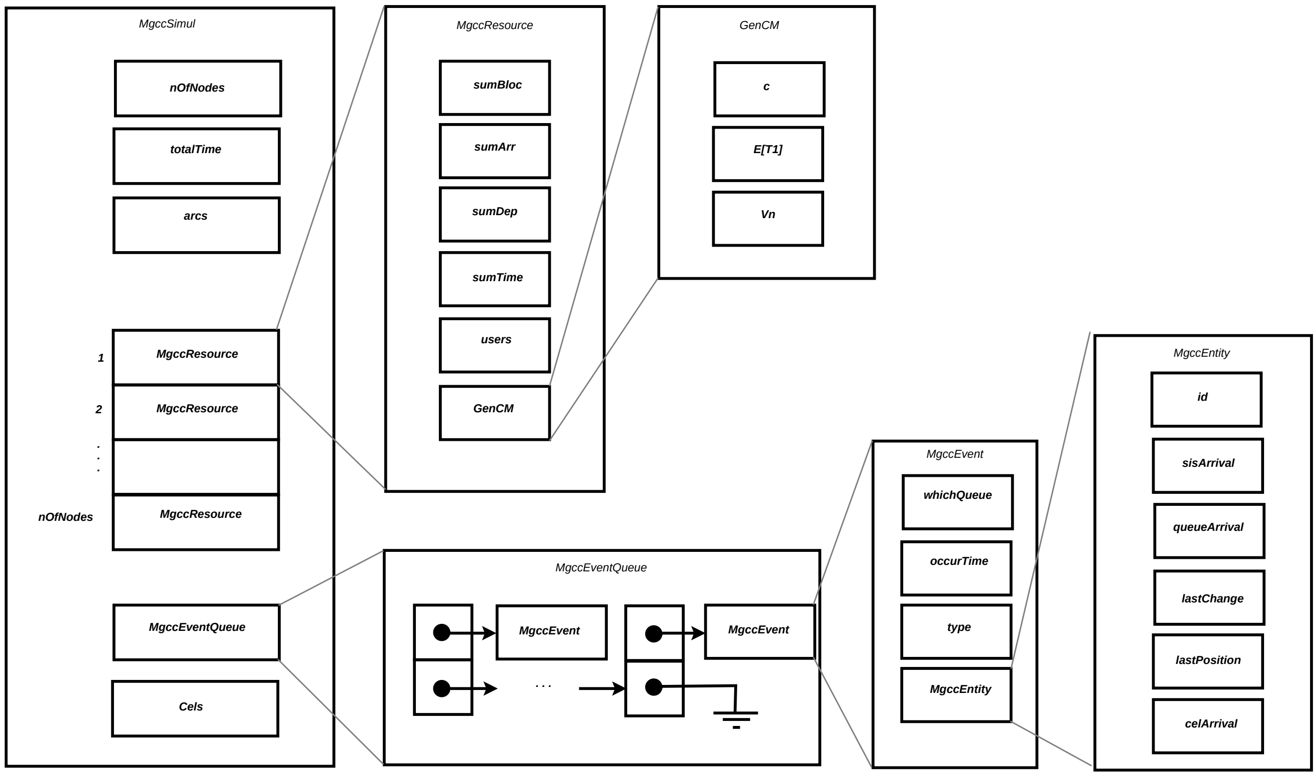

Figure 4: Object `MgccSimul`

run on the same PC with a 1.8 GHz Intel Pentium 4 CPU and 512 MB RAM, running Windows XP. The configurations were run for a three-hour simulation time, with the first hour being discharged for warm-up (see details in Robinson [21]), replicated 30 times for computing the descriptive statistics presented.

Four distinct basic configurations were tested. These were chosen because of their simplicity, the insights we can gain from them, and mainly because any complex configuration can be seen as a combination of these basic topologies. Of course, the combined effect of a certain combination of basic topologies will not be expected to be a perfect superimposition of the individual effects of these component topologies (remember that we do not have a linear system), but any insight we gain here may be helpful in analyzing the behavior of more complex networks, as we will see shortly.

One of the topologies is a basic series topology, presented in Fig. 6. Another is a basic split topology, seen in Fig. 7. A basic merge topology was also tested and is shown in Fig. 8. Finally, in order to better demonstrate the capabilities of the proposed model, a rather complex mixed topology was considered, which can be seen in Fig. 9. We shall now present and discuss the experimental results.

3.1 Series topology

In Fig. 6, we see a representation of a simplified cellular system composed of three cells, each of which has only one major transportation link, which we will model as an $M/G/c/c$ state-dependent queue. Without loss of generality, each transportation link is one kilometer in length and one lane in width. The descriptive statistics of the *time between departures* are shown in Tab. 1. It is remarkable to note the equivalence of the stochastic models for all three cells, for all tested arrival rates λ. However, the effect of the state-dependency is already noticeable, even for this simple case. In other words, up to the arrival rate of 4,000 vehicles per hour (veh/h), we observe an approximate equivalence between the averages and standard deviations for the *time between departures*. Above these values, the averages remain almost unchanged, around 1.85, because the system saturates, but the variability grows up to 14.3. Thus the arrival processes at cells #2 and #3, which are the departure processes from cells #1 and #2, respectively, no longer seem to be Markovian but rather resemble another hyper-exponential distribution. A practical relevant conclusion one could draw from these experiments is that if the load is high enough, an exponential model may not suffice at all for the arrivals, and state-dependent $M/G/c/c$ queueing networks may not be applicable.

In Fig. 10, we present histograms for the *time between departures*, for an arrival rate of 4,000 veh/h, which, from Tab. 6, seems to be the applicability limit of $M/G/c/c$ queueing network models. The adoption of an exponential model, for an arrival rate of 4,000, is visually sound.

In Figs. 11 and 12, a time-series plot of the *service times*

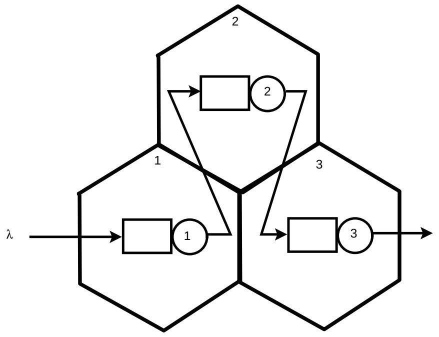

Figure 6: Three cells in series topology

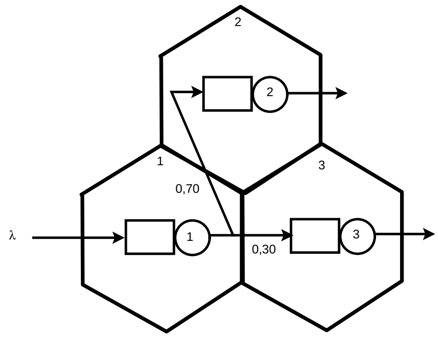

Figure 7: Three cells in split topology

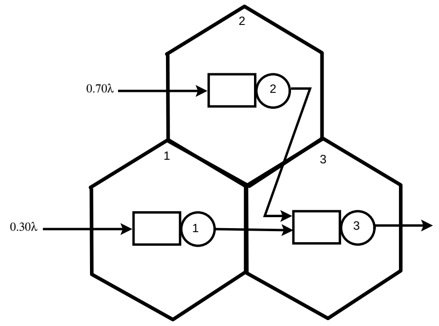

Figure 8: Three cells in merge topology

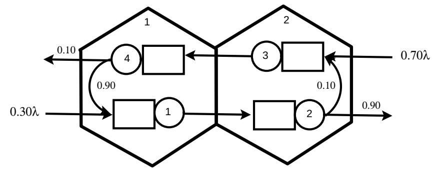

Figure 9: Two cells in a mixed topology

Table 1: Descriptive statistics of the average time between departures in series topology

λ	Cell	Min.	Q1	Median	Mean	StD	Q3	Max.
1,000	1	0.00	1.11	2.54	3.50	3.40	4.80	31.4
	2	0.00	1.10	2.47	3.46	3.35	4.68	28.6
	3	0.00	1.11	2.44	3.46	3.32	4.82	25.8
2,000	1	0.00	0.56	1.30	1.88	1.88	2.59	14.1
	2	0.00	0.56	1.29	1.87	1.86	2.59	15.3
	3	0.00	0.54	1.29	1.86	1.84	2.55	15.0
4,000	1	0.00	0.56	1.34	1.87	1.80	2.57	10.9
	2	0.00	0.56	1.33	1.87	1.80	2.60	11.1
	3	0.00	0.57	1.33	1.86	1.79	2.57	11.3
8,000	1	0.00	0.30	0.74	1.83	8.68	1.54	106
	2	0.00	0.30	0.74	1.83	8.73	1.51	106
	3	0.00	0.29	0.72	1.83	8.77	1.49	106
16,000	1	0.00	0.14	0.37	1.86	14.22	0.78	157
	2	0.00	0.14	0.37	1.86	14.24	0.78	159
	3	0.00	0.14	0.37	1.86	14.25	0.78	159

on cells is presented along with the corresponding histograms, for the arrival rates of 1,000 and 4,000 veh/h. Notice that for cellular system applications the *service time on cells* are equivalent to the *residency time on cells*, which is an important performance measure, as stressed earlier. We notice that exponential models are by no means appropriate for modelling such a random variable, as noticed by previous studies [11, 28].

3.2 *Split topology*

In Fig. 7, we show a cellular system in a simplified split configuration. In this case, each queue models a traffic link one kilometer long by one lane wide. The flow from link #1 divides into two links, #2 and #3, in the proportion of 70%-30%, respectively. Descriptive statistics of the *time between departures* are shown in Tab. 2. Again, under arrival rates up to 4,000 veh/h, exponential models seem to be applicable, as averages and standard deviations are similar. To visually attest for these conclusions, Fig. 13 shows histograms for the *time between arrivals*, with the adjustments for exponential models.

Table 2: Descriptive statistics of the average time between departures in split topology

λ	Cell	Min.	Q1	Median	Mean	StD	Q3	Max.
1,000	1	0.00	1.11	2.54	3.50	3.40	4.80	31.4
	2	0.00	1.39	3.35	4.70	4.46	6.75	32.3
	3	0.11	4.20	9.63	13.1	12.3	18.2	68.4
2,000	1	0.00	0.53	1.28	1.87	1.89	2.59	13.6
	2	0.00	0.72	1.83	2.70	2.73	3.84	26.0
	3	0.00	1.88	4.37	6.03	6.26	7.85	53.0
4,000	1	0.00	0.56	1.34	1.87	1.80	2.57	10.9
	2	0.00	0.80	1.88	2.65	2.55	3.53	16.6
	3	0.01	1.86	4.17	6.17	6.05	8.77	45.0
8,000	1	0.00	0.30	0.74	1.83	8.68	1.54	106
	2	0.00	0.35	1.07	2.72	8.93	2.42	88.3
	3	0.00	1.02	2.52	6.27	16.3	5.00	113
16,000	1	0.00	0.14	0.37	1.86	14.2	0.78	157
	2	0.00	0.19	0.52	2.68	15.9	1.26	161
	3	0.00	0.52	1.15	6.06	25.6	2.55	171

Figs. 14 and 15 present the behavior of the *service times on cells* (that is, the *residence times on cells*) and the corresponding histograms, for $\lambda = 1,000$ and 4,000 veh/h,

```
algorithm Simulate
        /* initialize event queue */
    Inicitialize(MgccEventQueue);
        /* create and insert 'last' event */
    MgccEvent ← new();
    MgccEvent.occurTime ← totalTime;
    MgccEvent.type ← end_simulation;
    Insert(MgccEventQueue,MgccEvent);
        /* create and insert 'first' events */
    for ∀n| λ_n ≠ 0 do
        MgccEvent ← new();
        MgccEvent.whichQueue ← n;
        MgccEvent.occurTime ← 0.0;
        MgccEvent.type ← arrival;
        Insert(MgccEventQueue,MgccEvent);
    end for
        /* simulate */
    MgccEvent ← GetNext(MgccEventQueue);
    while MgccEvent.type ≠ end_simulation do
        if MgccEvent.type = arrival then
            ProcessArrival(MgccEventQueue,MgccEvent);
        else if MgccEvent.type = departure then
            ProcessDeparture(MgccEventQueue,MgccEvent);
        else
            error, unknown event
        end if
        MgccEvent ← GetNext(MgccEventQueue);
    end while
    print results
end algorithm
```

Figure 5: Simulation algorithm

respectively. For $\lambda = 4,000$, the output of the $M/G/c/c$ state-dependent queueing network surprisingly indicates that the random variable *residence times on cells* has a very low variability. The important conclusion that can be drawn from these simulation results is that one will need to exercise extra care in adjusting some probability distribution for the random variable *service time*. The congestion that a given arrival rate might cause in the transportation link is hardly predictable without using a simulation tool such as the one used here, or some other analytical model.

3.3 *Merge topology*

The merge topology, Fig. 7, was considered only to attest to the symmetry of the results. One sees easily from Tab. 3 and Figs. 16, 17, and 18, that some symmetry is indeed present. In other words, nodes #3 and #1 behave exactly as nodes #1 and #3, respectively, in the split topology case. This behavior was expected and is an indication that the simulation model may be correct.

Table 3: Descriptive statistics of the average time between departures in merge topology

λ	Cell	Min.	Q1	Median	Mean	StD	Q3	Max.
1,000	1	0.01	3.32	7.52	11.7	12.3	17.2	114
	2	0.00	1.66	3.71	4.98	4.69	6.72	33.4
	3	0.00	1.05	2.49	3.47	3.24	4.85	23.1
2,000	1	0.02	1.86	3.99	5.82	5.47	8.20	31.3
	2	0.02	1.03	1.98	2.74	2.46	3.70	20.4
	3	0.02	0.66	1.34	1.86	1.71	2.40	10.4
4,000	1	0.00	0.96	2.46	3.74	3.69	5.19	29.4
	2	0.00	0.71	2.20	3.69	6.05	4.40	57.6
	3	0.00	0.44	1.21	1.86	2.13	2.38	14.7
8,000	1	0.03	0.54	1.59	3.68	6.22	4.03	45.5
	2	0.02	0.51	1.51	3.62	6.88	3.83	54.6
	3	0.02	0.41	1.09	1.83	2.67	2.02	17.4
16,000	1	0.01	0.25	0.98	3.66	9.55	2.35	79.4
	2	0.00	0.43	0.95	3.59	15.3	1.98	158
	3	0.00	0.18	0.50	1.81	6.12	1.34	65.2

3.4 *Mixed topology*

Finally, a mixed topology was considered. It is shown in Fig. 9, along with the corresponding routing probabilities and arrival rates. This complex topology was chosen to show that the simulation model is able to deal with more general cases rather than only with simple basic configurations. However, one must bear in mind that since the model is based on intensive simulation, the size of the manageable instances may be rather small. The simulation times may be prohibitive for large instances, but one could rely on the effectiveness of decomposition and aggregation techniques to reduce the size of real-life cases and make them tractable for simulation.

In this topology, each cell covers two transportation links rather than one. Each queue represents a link one kilometer long by one lane wide. From Tab. 4 and Fig. 19, one sees that an exponential model is fairly acceptable for the time between arrivals, if the arrival rate is not so high as to saturate the system (in this case, for $\lambda \leq 4,000$ veh/h).

Table 4: Descriptive statistics of the average time between departures in a mixed topology

λ	Cell	Min.	Q1	Median	Mean	StD	Q3	Max.
1,000	1	0.00	0.92	2.26	3.19	3.17	4.37	19.5
	2	0.00	0.57	1.43	2.02	1.99	2.83	12.6
2,000	1	0.00	0.47	1.20	1.69	1.70	2.35	17.4
	2	0.00	0.32	0.77	1.08	1.04	1.53	7.56
4,000	1	0.00	0.45	1.11	1.57	1.61	2.26	13.6
	2	0.00	0.31	0.74	0.96	0.89	1.35	6.85
8,000	1	0.00	0.36	1.07	1.58	1.96	2.00	17.7
	2	0.00	0.26	0.67	0.95	1.06	1.32	12.2
16,000	1	0.00	0.21	0.69	1.54	3.29	1.62	33.1
	2	0.00	0.17	0.43	0.93	2.47	0.86	33.1

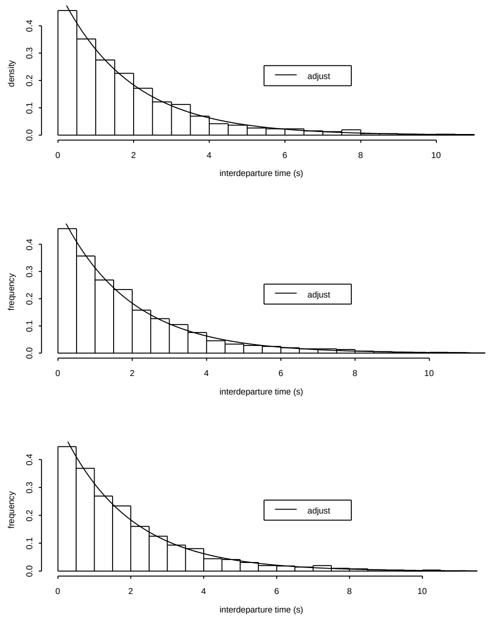

Figure 10: Time between departures in series topology for $\lambda = 4,000$

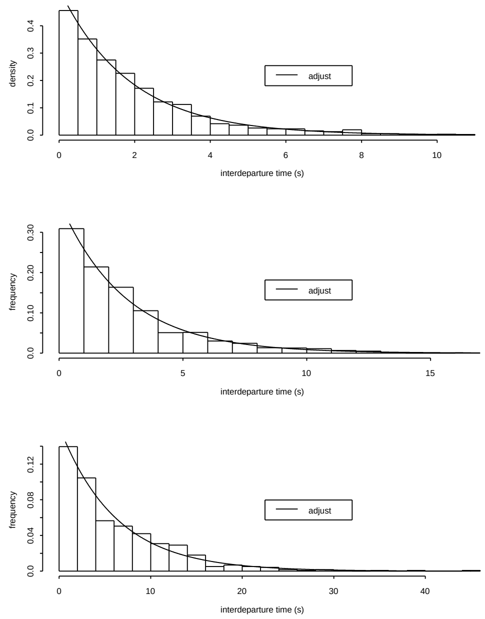

Figure 13: Time between departures in split topology for $\lambda = 4,000$

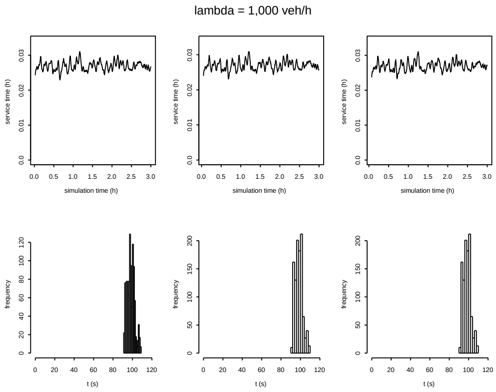

Figure 11: Service time in series topology for $\lambda = 1,000$

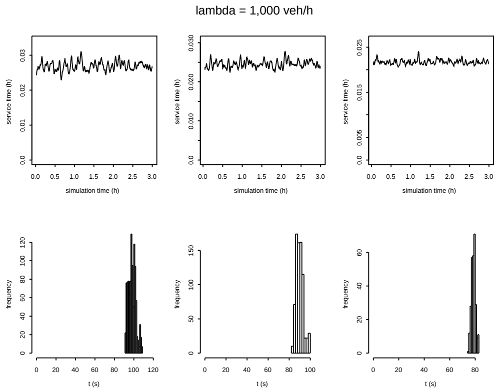

Figure 14: Service time in split topology for $\lambda = 1,000$

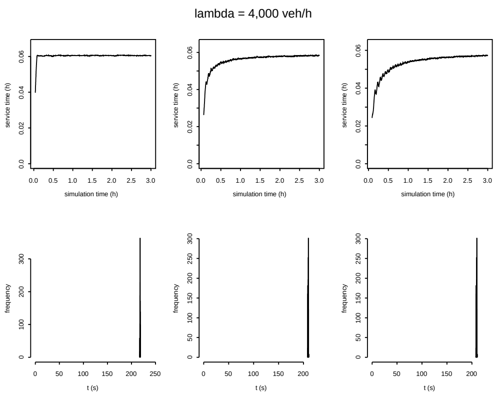

Figure 12: Service time in series topology for $\lambda = 4,000$

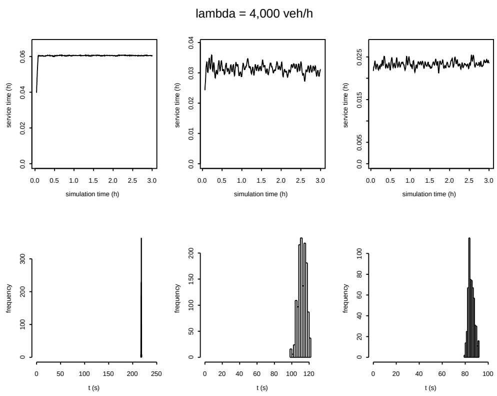

Figure 15: Service time in split topology for $\lambda = 4,000$

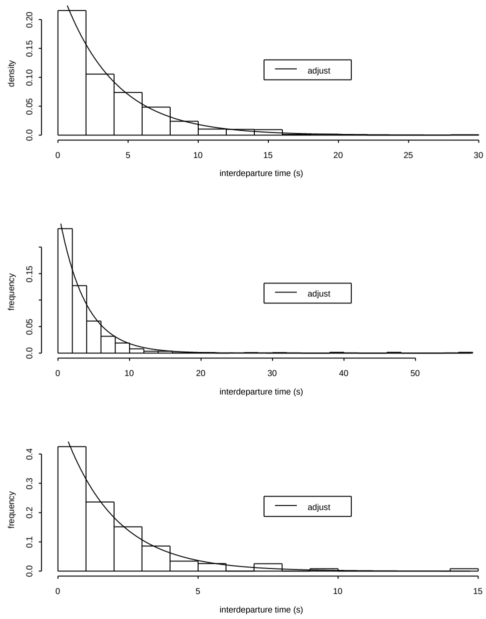

Figure 16: Time between departures in merge topology for $\lambda = 4,000$

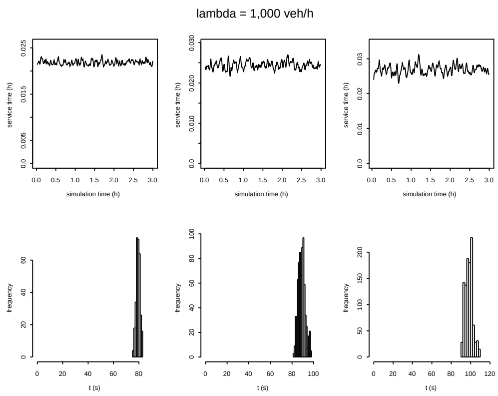

Figure 17: Service time in merge topology for $\lambda = 1,000$

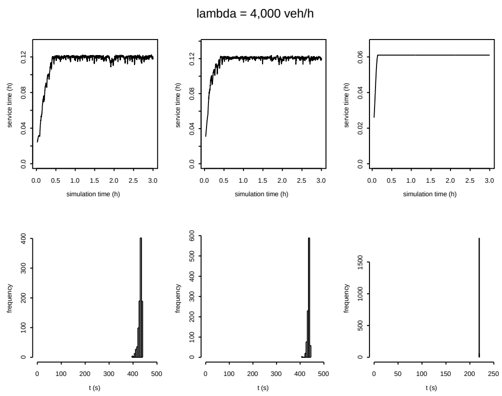

Figure 18: Service time in merge topology for $\lambda = 4,000$

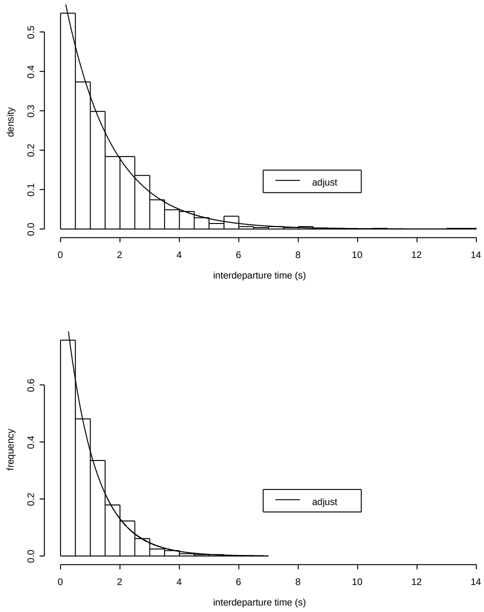

Figure 19: Time between departures in mixed topology for $\lambda = 4,000$

It is when one analyzes the *residence time on cells* that the most curious result appears. Bimodal distributions emerge, as seen in Figs. 20 and 21. From these results, it is clear that sometimes the analyst will need to use mixtures of probability distributions for the random variable *residence time on cells*, rather than any single model.

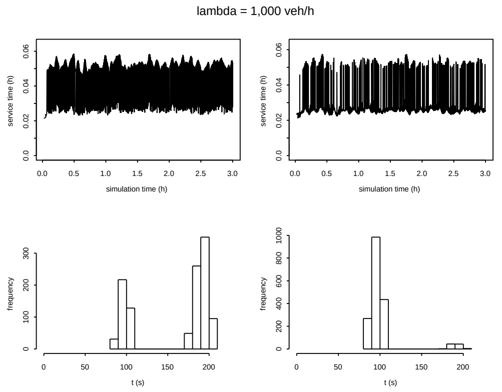

Figure 20: Service time in mixed topology for $\lambda = 1,000$

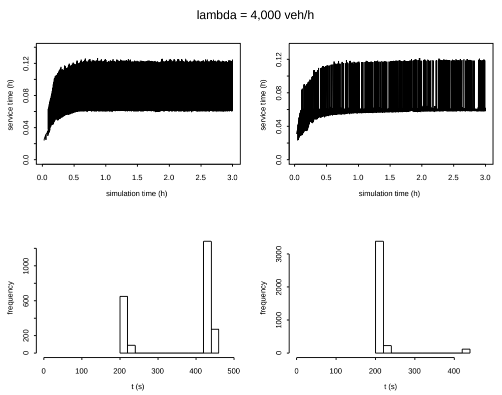

Figure 21: Service time in mixed topology for $\lambda = 4,000$

4 Conclusions and final remarks

A novel approach was proposed to model mobility in communication networks, based on finite state-dependent queueing models. This stochastic model was applied successfully in the past for vehicular and pedestrian traffic problems, as well as in manufacturing system modeling. Basically, this new approach does not use anything conceptually new in modeling vehicular traffic, since it merely considers the reduction of the average speed (service rate), with the increase in density of users in the system. However, to the best knowledge of the authors, such a strongly intuitive key concept has not previously been used explicitly in modeling users on mobile communication networks. With the aim of stressing the impact of including the state dependency on the mobility models, we present a newly-developed discrete-event simulation model and highlighted some of its performance measures.

Among the main insights garnered from the simulations, we note that, contrary to the belief of some researchers, the arrival process at the cells may not adhere to a Markovian process, under heavily loaded mobile systems, as happens often in large cities. Additionally, the simulation studies confirmed that the residence times on cells are not truly exponential, and it may be risky to consider any other probability distribution without a careful evaluation of the congestion status of the cell. Finally, in complex topologies, one may even find multi-modal probability distributions for the residence time on cells.

Some questions were answered by this research, but the results presented give rise to many others. Firstly, it is unknown what the maximum size of the manageable instance is in this simulation model. We do not expect it to be too large, as simulation usually tends to be time-consuming. Although someone could argue that one could use decomposition and aggregation techniques, as an approximation to complex systems, it would be of benefit to develop *ad-hoc* analytical models. We were only concerned here about the influence of state-dependent stochastic models in one random variable, namely, the *residence time on cells*. Actually, using finite state-dependent queueing models is only the beginning. Once this has been established, it is important to consider the effects, for instance, on the traffic of calls, the number of handoffs, call duration, and overall call performance, which are completely unknown. Additionally, another interesting area of research is capacity allocation. Indeed, successful results have been reported for finite queueing networks in manufacturing systems [4], for which a minimum capacity must be set while ensuring a certain quality of service, described in terms of low blocking probabilities.

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