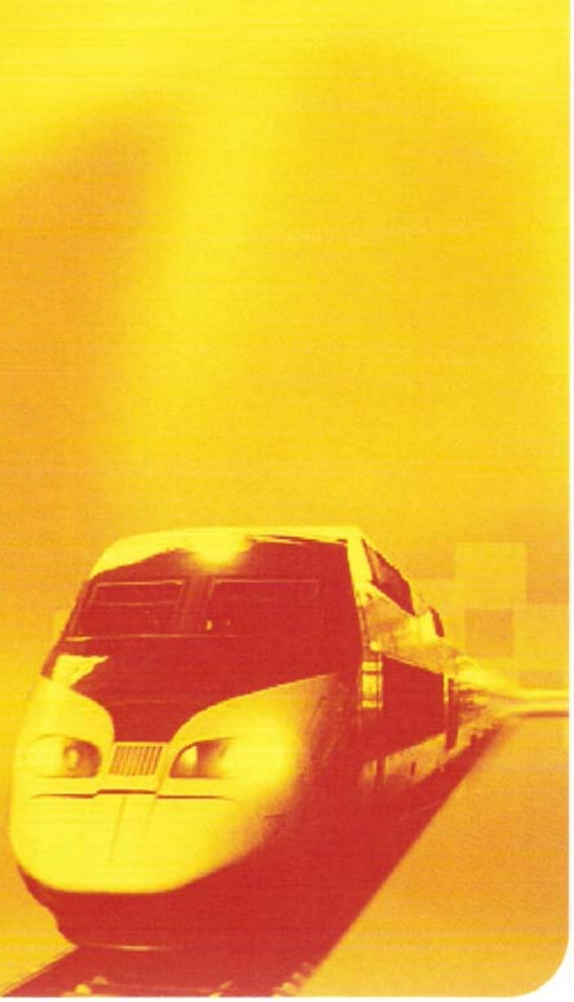
Volume III

PEITS 2010

2010 3rd International Conference on Power Electronics and Intelligent Transportation System

November 20-21, 2010, Shenzhen, China



IEEE

2010 3rd International Conference on Power Electronics and Intelligent Transportation System

Traffic Assignment Under State-Dependent Travel Times*

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***Abstract*— In this paper we investigate an $M/G/c/c$ state-dependent queueing network formula in the context of a System Optimum (SO) problem. The SO formulation is equivalent to a situation in which users cooperate with each other in order to minimize the overall travel cost, which is usually expressed in terms of travel times. Differently from the classical travel cost formulas, the expression used here is not convex but S-shaped. Consequently, multiple solutions may be present for the SO, which justifies the use heuristic procedures such as the Differential Evolution (DE) algorithms. Preliminary results from computational experiments with an DE are given to attest for the efficiency of the approach and the quality of the solutions given.**

***Index Terms*— System optimum, state-dependent, queues, queueing networks.**

I. Introduction

In the classical Wardrop System Optimum (SO) assignment model, the users are assumed to cooperate with each other in order to minimize the overall travel costs [1]. Even though the SO is based on a rather non-realistic behavioral assumption, we argue that its solution may be seen as a result of a well-succeeded control action on the transportation network. In other words, signal timings may be re-optimized and alternative routes may be re-defined in response to an increase in demand causing extra green light timings, as it is known that traffic lights and routing can improve the flow, depending on the traffic densities (e.g., Dynamic Routing Information Panel Systems - DRIPS).

One major problem is that the overall travel costs, usually expressed in term of time, are based on deterministic travel times on the single links, yet these times are known to be rather variable between trips, within and between days etc. Usually, only the mean travel times are represented in the SO model and an important issue in urban transportation networks is to model congestion, especially during rush hours, when the demand exceeds capacity. As seen in Figure 1, which presents results from many empirical studies for North American roads [2]–[6], congestion may be perceived as a decrease in the mean

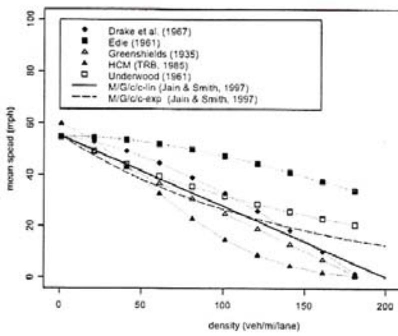

Fig. 1. Empirical Distributions for Vehicular Traffic Flows [2]–[6] and $M/G/c/c$ State-dependent Models [9]

speed when the vehicular density increases, resulting in the well-known speed-flow density curves (see the seminal work by Greenshields [4]).

There have been successful attempts in the literature to model how users select their route in a congested network [7]. Under the assumption that drivers have perfect knowledge about travel costs, they will choose the best route according to Wardrop's first principle, which is equivalent to the mixed-strategy Nash equilibrium of n-players, non-cooperative game [8]. This leads to the Deterministic User Equilibrium (UE), another classical traffic assignment model [1]. In the equilibrium, routes carrying a positive flow will have equal travel costs. Unfortunately the conclusion is that, by doing so, scarce resources (street and road capacity) are used in an inefficient way [7]. For this reason, the focus here will be in the SO model.

In this paper, we discuss an extension of the SO model by applying the state-dependent $M/G/c/c$ queueing network model in order to estimate the travel times, usually the main factor for route selection. The $M/G/c/c$ state-dependent model has been used in the past [9]. In fact, Cruz et al. [10] have shown recently that the $M/G/c/c$ state-dependent model is also quite effective for modeling the travel time on single links, in comparison to well-established formulas, because of its capability of representing traffic congestion, as it is seen in Figure 1.

Also discussed in this paper is a successful way to solve the SO, by means of a finite state-dependent queueing network approach and by using a Differential Evolution (DE) heuristic, which is part of the family of Genetic Algorithms (GA). The hybrid modeling approach (queueing networks and Differential Evolution) results in efficient and acceptable solutions for the problem on hand.

In Section II the mathematical programming formulation

*This research has been partially funded by CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico; grants 201046/1994-6, 301809/1996-8, 307702/2004-9, 472066/2004-8, 304944/2007-6, 561259/2008-9, 553019/2009-0, 550207/2010-4, 501532/2010-2, 303388/2010-2), by CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior; grant BEX-0522/07-4), and by FAPEMIG (grants CEX-289/98, CEX-855/98, TEC-875/07, CEX-PPM-00401/08, and CEX-PPM-00390-10).

978-1-4244-9162-9/10/$26.00 ©2010 IEEE

TABLE I
BASIC NETWORK NOTATION

Variable	Description
$\mathcal{N}$	node (index) set
$\mathcal{A}$	arc (index) set
$\mathcal{R}$	set of origin nodes; $\mathcal{R} \subseteq \mathcal{N}$
$\mathcal{S}$	set of destination nodes; $\mathcal{S} \subseteq \mathcal{N}$
$\mathcal{K}_{rs}$	set of paths connecting origin-destination (O-D) pair r-s; $r \in \mathcal{R}$, $s \in \mathcal{S}$;
x_a	flow on arc a; $\mathbf{x} = (\ldots, x_a, \ldots)$
$c_a(x_a)$	travel time on arc a; $\mathbf{c}(\mathbf{x}) = (\ldots, c_a(x_a), \ldots)$
f_k^{rs}	flow on path k connecting O-D pair r-s; $\mathbf{f}^{rs} = (\ldots, f_k^{rs}, \ldots)$; $\mathbf{f} = (\ldots, \mathbf{f}^{rs}, \ldots)$
c_k^{rs}	travel time on path k connecting O-D pair r-s; $\mathbf{c}^{rs} = (\ldots, c_k^{rs}, \ldots)$; $\mathbf{c} = (\ldots, \mathbf{c}^{rs}, \ldots)$
q^{rs}	trip rate between origin r and destination s; $(\mathbf{q})^{rs} = q^{rs}$
$\delta_{a,k}^{rs}$	indicator variable: $\delta_{a,k}^{rs} = \begin{cases} 1, & \text{if link } a \text{ is on path } k \text{ between } O\text{-}D \text{ pair } r\text{-}s, \\ 0, & \text{otherwise;} \end{cases}$ $(\mathbf{\Delta}^{rs})_{a,k} = \delta_{a,k}^{rs}$; $\mathbf{\Delta} = (\ldots, \mathbf{\Delta}^{rs}, \ldots)$

for the classical SO assignment model is presented in detail. Section III describes the algorithm for solving the SO, as well as the performance evaluation algorithm. Section IV focuses on the computational experiments. Finally, Section V summarizes the paper.

II. Mathematical Programming Formulations

Well-known from the literature, the System Optimum (SO) formulation is briefly reviewed as follows. The network notation used is summarized in Table I. The SO formulation is equivalent to a situation in which users cooperate with each other in order to minimize the total travel costs. According to Wardrop's second principle, the SO model is formulated as follows:

$$\min z(\mathbf{x}) = \sum_a x_a c_a(x_a),$$

s.t.:

$$\begin{aligned} \sum_k f_k^{rs} &= q^{rs}, && \forall\, r, s, \\ x_a &= \sum_r \sum_s \sum_k f_k^{rs} \delta_k^{rs}, && \forall\, a, \\ f_k^{rs} &\geq 0, && \forall\, k, r, s, \end{aligned}$$

where x_a is the flow on link a, $c_a(x_a)$ is the travel cost on link a, f_k^{rs} is the flow on route k between origin r and destination s, and q^{rs} is the demand between r and s. The complete notation is seen in Table I.

Notice that the travel costs, $c_a(x_a)$, are usually expressed in terms of times and are usually given from classical formulas (BPR-like [11]) that have been constructed over the past 40 years for this purpose of describing the travel time (e.g., [12]). However, as one shall see shortly, none of these formulas is able to represent congestion as the $M/G/c/c$ state-dependent queueing model does.

III. Algorithms

The SO and similar models have been solved by researchers by many algorithms. Exact algorithms have been proposed as well as heuristics including the differential evolution algorithm itself [10]. Basically, what we have to

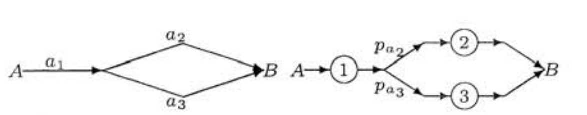

Fig. 2. Three-road Network and Corresponding $M/G/c/c$ Model

do is to find routing probabilities for alternative routes to go from A to B (e.g., p_{a_2} and p_{a_3}, as seen in Figure 2), in such a way that the objective function $\sum_a x_a c_a(x_a)$ is minimized. The steps are detailed as follows.

A. Differential Evolution Algorithm

A Differential Evolution (DE) algorithm will be used to compute the optimal routing probability vector, $\mathbf{p} = (\ldots, p_a, \ldots)$, $\forall a \in \mathcal{A}$. DE algorithms are part of a broader family of Genetic Algorithms (GA). Some of the features of DE algorithms that justify their use for solving SO problems include fastness, robustness, ease of use, and ability to operate on flat surfaces. Additionally, Babu and Sastry [13] found that DE algorithms were the best evolutionary computation method after the study of seven difficult design and control MINLP problems in chemical engineering.

A DE algorithm is defined as a parallel direct search method which operates on a population P_G of constant size that is associated with each generation G and consists of NP vectors, or candidate solutions, $\mathbf{X}_{p,G}$, $p = 1, 2, \ldots, NP$. Each vector $\mathbf{X}_{p,G}$ consists of D decision variables $X_{o,p,G}$, $o = 1, 2, \ldots, D$. This is briefly summarized as:

$$\begin{aligned} P_G &= \{\mathbf{X}_{1,G}, \mathbf{X}_{2,G}, \ldots, \mathbf{X}_{p,G}, \ldots, \mathbf{X}_{NP,G}\}, \\ \mathbf{X}_{p,G} &= \{X_{1,p,G}, X_{2,p,G}, \ldots, X_{o,p,G}, \ldots, X_{D,p,G}\}, \\ G &= 1, \ldots, G_{\max}, \\ NP &\geq 4. \end{aligned}$$

Each routing probability is then considered as the decision variable, $X_{o,p,G} \equiv p_a$. More details on the DE algorithms, including mutation schemes, values for the control parameters, other constraint handling methods, and stopping criteria may be found in literature [14].

B. Computation of the Travel Times

Single Queues

First, a single road link may be seen as c parallel servers to its occupants and c is also the maximum number of users simultaneously allowed in the system. Consequently, there is no buffer (waiting space). Second, based on the empirical results presented in Figure 1, the service times depend on the number of users currently in the system. As a consequence, an $M/G/c/c$ state-dependent queueing model seems to be a reasonable tool to describe a single link [9]. The limiting probabilities $p_n \equiv \Pr[N = n]$, for the random number of entities N in an $M/G/c/c$ state-dependent queueing model will be:

$$p_n = \left\{ \frac{\left[\lambda E[T_1]\right]^n}{n! f(n) f(n-1) \cdots f(2) f(1)} \right\} p_0, \qquad (1)$$

where $n = 1, 2, \ldots, c$. The empty system probability, p_0, is given by:

$$p_0^{-1} = 1 + \sum_{i=1}^{c} \left\{ \frac{\left[\lambda E[T_1]\right]^i}{i! f(i) f(i-1) \cdots f(2) f(1)} \right\}, \quad (2)$$

where λ is the arrival rate and $E[T_1] = l/V_1$ is the expected service time of a lone vehicle in a link of length l, considering that V_1 is the speed of a lone occupant. The capacity, c, is given by:

$$c = \lfloor klw \rfloor,$$

where l is the length of the link, w is its width (in number of lanes), k is its capacity (per length-unit per lane), and $\lfloor x \rfloor$ is the largest integer not superior to x.

Considering vehicular related applications, k represents the jam density parameter (in veh/mi-lane). Normally k ranges from 185-265 veh/mi-lane. Additionally, $f(n) = V_n/V_1$ is the service rate, that is, the ratio of the average speed of n users in the link to that of a lone occupant, V_1.

Now the aim is to derive a congestion model that represents the effect depicted in Figure 1, i.e. a service rate that decays with an increase in the number of users in the system. The following exponential model seems quite reasonable:

$$f(n) = \exp\left[-\left(\frac{n-1}{\beta}\right)^{\gamma}\right],$$

with

$$\gamma = \log\left[\frac{\log(V_a/V_1)}{\log(V_b/V_1)}\right] / \log\left(\frac{a-1}{b-1}\right),$$

and

$$\beta = \frac{a-1}{[\log(V_1/V_a)]^{1/\gamma}} = \frac{b-1}{[\log(V_1/V_b)]^{1/\gamma}}.$$

The values a and b are arbitrary points used to adjust the exponential curve. In vehicular applications, common values are $a = 20lw$ and $b = 140lw$, which correspond to the densities of 20 and 140 veh/mi-lane respectively. Looking at the curves presented in Figure 1, reasonable values are $V_a = 48$ mph and $V_b = 20$ mph.

From (1), important performance measures can be derived

$$\begin{cases} p_c &=& \Pr[N = c], \\ \theta &=& \lambda(1 - p_c), \\ L &=& E[N] = \sum_{n=1}^{c} n p_n, \\ W &=& E[T] = L/\theta, \end{cases}$$

where p_c is the blocking probability, $\theta \equiv x_a$ is the throughput in veh/h, L is the expected number of customers in the link (also known as work-in-process, WIP), and $W \equiv c_a(x_a)$ is the expected service time in hours (here derived from Little's formula).

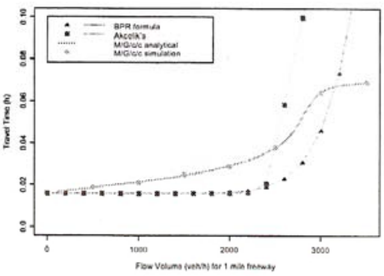

Fig. 3. One-mile Vehicular Traffic Flows [10]

Networks of Queues

We remind that so far the performance evaluation is an issue only half solved. Indeed, deriving performance measures for *networks* of $M/G/c/c$ state-dependent queues is a task considerably more complex. The input in each queue is a function of the routing probabilities which defines the overall performance along with the inter-blocking effects. The Generalized Expansion Method (GEM) is a technique to approximately evaluate the performance of finite queueing networks. The method was proposed by Kerbache and Smith [9] and is a combination of repeated trials and node-by-node decomposition approximation methods. Further details will not be given here but can be found easily in the literature [9]. Equation (3) represents the final step of the GEM, which ultimate goal is to provide an approximation scheme to update the service rates μ_i of upstream nodes that takes into account all blocking after service caused by downstream nodes j:

$$\tilde{\mu}_i^{-1} = \mu_i^{-1} + p_{c_j}(\mu_h')^{-1}. \quad (3)$$

Final Remarks

In conclusion, for a given routing probability vector, p, it should not be hard to estimate the corresponding performance measure, $\sum_a x_a c_a(x_a)$, which is the objective function to be minimized. The problem is that the use of state-dependent travel times complicates the optimization problem. In fact, as seen in Figure 3, typical travel time functions (BPR-like) differ quite much from $M/G/c/c$ state-dependent queueing model functions, specially under heavy traffic. Under low traffic, the queueing approach is close to the classical formula but the $M/G/c/c$ model predicts S-shaped travel-time curves, which represent serious trouble for any optimization algorithm.

IV. Computational Experiments

The algorithm was coded in C++ and is available from the authors upon request for research and educational purposes. The experiments were conducted in a PC, under Windows Vista operating system. The example illustrated in Figure 2 is a three-link network. A and B are connected by link a_1 and two alternative links, a_2 and a_3, where one of the alternative routes is longer (and consequently slower) than the other. The adjustments for the $M/G/c/c$ model is presented in Table II. The algorithm was run for different arrival rates (λ) and the results obtained may be seen in Table III.

TABLE II

SETTINGS FOR THE THREE-ROAD NETWORK

Route	Length*	Width†	$V_1^{\ddagger}$	$V_a^{\ddagger}$	$V_b^{\ddagger}$	c (veh)	$E[T_1]^{\sharp}$
a_1	0.80 (0.50)	5	25 (40)	23 (37)	10 (16)	800	0.0320 (115)
a_2	2.50 (1.55)	2	20 (32)	18 (29)	6 (10)	1,000	0.1250 (450)
a_3	1.85 (1.15)	2	20 (32)	18 (29)	6 (10)	740	0.0925 (333)

Remarks: *in miles (km); †in # lanes; ‡in mph (km/h); ♯in h (s);

TABLE III

OPTIMAL ASSIGNMENTS FOR THE THREE-ROAD NETWORK

λ	route	assignment	$E[T]^*$
0	a_1-a_2	n/a	0.1570 (565)
	a_1-a_3	n/a	0.1245 (448)
1,000	a_1-a_2	370	0.1635 (588)
	a_1-a_3	630	0.1341 (483)
2,000	a_1-a_2	890	0.1791 (645)
	a_1-a_3	1,110	0.1484 (534)
4,000	a_1-a_2	1,496	0.4742 (1,707)
	a_1-a_3	**1,225**	**0.8964 (3,227)**
8,000	a_1-a_2	1,507	0.4750 (1,710)
	a_1-a_3	**1,224**	**0.8970 (3,229)**

*in hours (in seconds);

When there is no arrivals (rate is zero), we have that the expected travel time is of a lone occupant, which is the sum of the lone occupant expected travel times of the respective links. From $\lambda = 1,000$, we observe an increase on the expected travel time caused by the congestion level. Notice that the expected service times for both routes are never equal which means that users with knowledge about the travel costs could reduce their own travel time by changing from a slow to a fast route. Such improvements would be impossible for an UE optimum solution (see Sheffi [1]) but not for the SO. In fact, the SO model seeks *overall* (instead of individual) minimum costs.

Concerning the optimum assignments, we remark that the traffic is mostly directed towards the fastest route (i.e., the route with the lowest expected travel time) and then the remaining traffic goes through the slowest route. This is what it should be expected, which is encouraging. For this network, we also observe that up to the arrival rate of 2,000 users per time unit, all the traffic goes through the network without any blocking (i.e., roughly the sum of the assignments equals the arrival rate). However, above this critical rate the network seems to have reached its capacity because only a fraction of the additional traffic can successfully go through.

Finally we would like to notice a somewhat unexpected behavior at link a_1-a_3, that is, an increase on its travel time (from 0.8964 to 0.8970) in spite of a reduction of the traffic on it (from 1,225 to 1,224). This is a type of behavior that would be impossible under BPR-like travel time estimation formulas but that is perfectly reasonable under $M/G/c/c$ state-dependent models. Indeed, as reported by many researchers the $M/G/c/c$ state-dependent models induce a throughput-*vs.*-arrival curve that reaches a maximum, decreases a bit, and then stabilizes (see Figure 4).

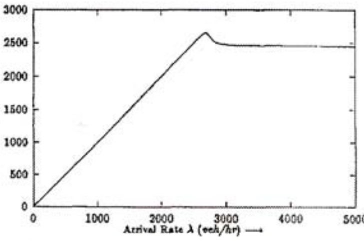

Fig. 4. Exponential Congestion [9]

V. CONCLUDING REMARKS

The System Optimum (SO) problem was solved under a different stochastic travel time formula which is based on $M/G/c/c$ state-dependent queues. This new formula clearly has advantages over some expressions that do not properly represent the congestion (i.e., the travel times should be reduced with an increase link congestion). On the other hand, the resulting travel time function is S-shaped, which brings difficulties to the optimization algorithms as now local optima may be present. Computational results attest that Differential Evolution (DE) heuristics may be effective in solving the SO problem, as sound solutions were found. Additionally, the solutions seemed to be robust as demonstrated by our computational experiments.

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IEEE Catalog Number: CFP1075E-PRT
ISBN: 978-1-4244-9162-9

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