Universidade Federal de Minas Gerais
Instituto de Ciências Exatas
Departamento de Estatística

Design of Economically Optimal Zero-Defect Acceptance Sampling with Rectification when Diagnosis Errors are Present

Roberto C. Quinino, Emílio Suyama,
e Linda L. Ho

Relatório Técnico
RTP-01/2004
Série Pesquisa

DESIGN OF ECONOMICALLY OPTIMAL ZERO-DEFECT ACCEPTANCE SAMPLING WITH RECTIFICATION WHEN DIAGNOSIS ERRORS ARE PRESENT

Roberto da Costa Quinino & Emílio Suyama
Departamento de Estatística – ICEX - UFMG
e-mail: roberto@est.ufmg.br & suyama@est.ufmg.br

Linda Lee Ho
Departamento de Eng. de Produção –EP-USP
e-mail: lindalee@usp.br

Abstract

In this paper we present the optimum sampling size in zero-defect acceptance sampling with rectification under diagnosis errors. Its development is based on an economical model. The procedures are implemented in a program using the software Matlab and illustrated by an example.

Key-words: Zero-defect acceptance sampling, diagnosis errors, cost function, rectification.

1. Introduction

Let us consider that items are manufactured and evaluated by attributes using a well-known tool: the acceptance sampling. In T lots each one with N products, a sample of m is extracted from each lot. If all sampled items are conforming in the inspection, the lot is accepted. Otherwise all items (sampled and non-sampled) are inspected, rectified the non-conforming ones and then the lot is accepted. Such procedure is known as zero-defect sampling with rectification. Rectification, i.e. replacing or discarding all non-conforming units after 100% inspection of rejected lots, is frequently used when manufacturing costs are high. The most common application of such acceptance sampling scheme is in the semiconductor manufacturing. Figure 1 illustrates such procedure.

Some papers about zero-defect sampling with rectification can be found in the literature. We may mention the contributions from Hahn (1986), Brush *et al*. (1990), Greenberg & Stokes (1992) and Anderson *et al*. (2001). In these papers, the main objective is to present estimator for the number of non-conforming items in such sampling scheme. In Anderson *et al*. (2001), they introduced the possibility of the classification criteria in zero-defect sampling with rectification may present diagnosis errors. That is, one item is evaluated as non-conforming but in reality it is conforming, or an item is classified as conforming but it is non-conforming. About diagnosis errors, many authors have made contributions on this subject. For example, Johnson *et al.* (1991) have early pointed out that the diagnosis errors can endanger the performance of an acceptance sampling. Minton (1972) provides expressions to analyze the effect of inefficient inspection and correction on the power of single sampling inspection plans, mainly in misclassifications of defectives as non-defectives. Different authors have presented methodologies to minimize the impact of diagnosis errors in the acceptance sampling. We may list Greenberg & Stokes (1995), Markowski & Markowski (2002), Quinino & Ho (2003) and Quinino & Suyama (2002).

The use of economical model to plan acceptance sampling is not a new subject, but it is still a subject of great interest as mentioned Wetherril & Chiu (1975). It was recently used in Ferrel & Chhoker (2002) to determine the producer's tolerance that minimizes producer's loss and consumer's loss in a single sampling, with inspection and non-inspection procedures using a quadratic function to describe the consumer's cost. Aminzadeh (2003) actually used the Inverse Gaussian distribution as a lifetime model to obtain optimal values for sample size and action limit for employing economic variable acceptance-sampling plans based on step-loss function. Starbird (1997) derive the conditions under which zero-defect is the policy that minimizes the supplier's expected annual cost.

In this paper, we will consider the determination of an economically optimum sample size m that minimizes a cost function in zero-defect acceptance sampling with rectification procedure. The components of such function include the inspection cost, costs due to the presence of non-conforming items in accepted lots and costs due to diagnosis errors. The inclusion of diagnosis errors in the sample size determination in such sampling inspection procedure is a natural extension of the earlier papers mentioned. Economical models mentioned in the literature do not include the possibility of the diagnosis errors and rectification.

In Section 2, we introduce the notation and hypothesis considered in this paper. The expected cost function and the procedure to determine the optimum value of m is developed in Section 3. As this probabilistic model of sampling process can be viewed as a Markov chain, the description of the absorbent and /or transitory states and their transition matrices is presented in Section 4. This procedure is illustrated by a numerical example in Section 5 and we finish this paper with discussions and extensions in future works.

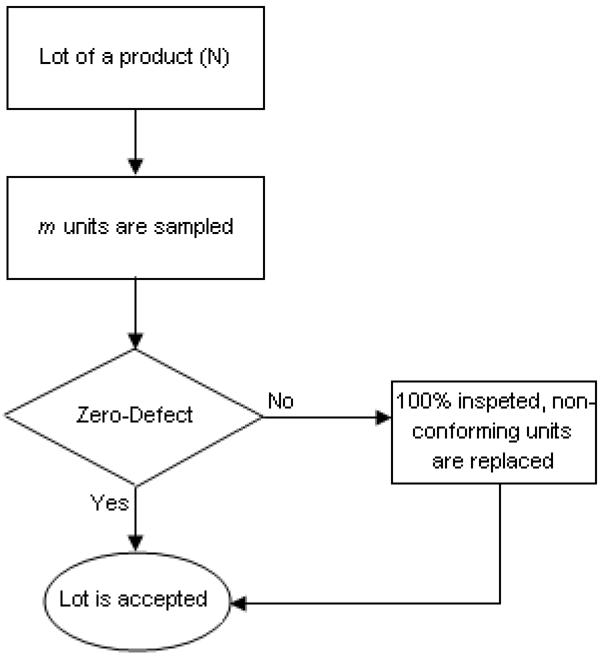


Figure 1: Acceptance sampling: zero defect with rectification

2. Notation and hypothesis

Consider that from a lot with N units, a random sample of m units is selected without replacement. Let D be the number of non-conforming units in the lot. D is a binomial random variable (N, p), with probability π, or D is equal to zero with probability $(1-\pi)$. This family of distributions is sufficiently flexible to give a good fit to observed distributions of lot quality by appropriate choice of the probability π, and it allows a simple interpretion and leads to a simple theory [Hald (1981)].

Let:

$e_1 \rightarrow$ the probability of a conforming item being classified as non-conforming;
$e_2 \rightarrow$ the probability of a non-conforming item being classified as conforming;
$c_0 \rightarrow$ the cost to inspect an item ;
$c_1 \rightarrow$ the cost of a non-conforming item in the accepted lot;
$c_2 \rightarrow$ the cost to judge erroneously an item as non-conform when it is conforming ;
$D_1 \rightarrow$ the number of actual non-conforming items in the initial sample of size *m* in the lot;
$D_2 \rightarrow$ the number of actual non-conforming items in (*N*-*m*) non-sampled items in the lot;
$D = D_1 + D_2 \rightarrow$ the number of non-conforming items in the lot;
$Y_1 \rightarrow$ the number of items declared non-conforming after inspection in the initial sample of size *m* in the lot;
$Y_2 \rightarrow$ the number of items declared non-conforming in (*N*-*m*) non-sampled items in the lot if the lot were rectified;
$Y = Y_1 + Y_2 \rightarrow$ the number of items that would be declared as non-conforming in lot if the lot were rectified;
$D_1|D \rightarrow$ the conditioned distribution of D_1 on D and it follows a *Hypergeometric (m, D, N)* .

3. Cost function

In this Section, the expected cost function per lot (E_m) is developed employing the earlier notations and hypothesis from Section 2. The expected cost function is composed by three parts. The first one (E_m^1) is related to costs of inspection of *m* items and the possibility to inspect the *(N-m)* non-sampled items. Such event is conditioned to the classification of at least one non-conforming item in the *m* initial inspected items and the probability of this event is denoted by $P(Y_1 > 0)$:

$$E_m^1 = c_0 m + c_0 (N-m) P(Y_1 > 0).$$

The second component (E_m^2) is due to the possibility of an item being classified as conforming when it is non-conforming item. Such result can produce differences in the expenses when the lot is accepted or when it is rejected in the inspection. Figure 2 illustrates such procedure.

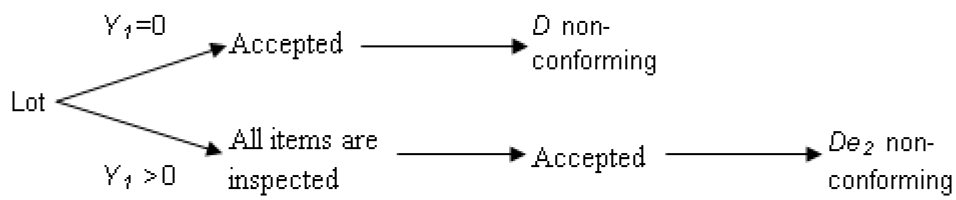


Figure 2: non-conforming when the lot is accepted or when it is rejected

Thus, we have

$$E_m^2 = c_1 E\left[I_{[Y_1=0]} D + e_2 I_{[Y_1>0]} D \right]$$

where $I_{[\bullet]}$ denotes an indicator function; $E(\bullet) \rightarrow$ denotes the expected value of a random variable. The last part (E_m^3) is due to the consequence of classifying an item as non-conforming when it is a conforming item. In this case, the lot is rejected and consequently all items classified as non-conforming and there is a chance to be rectified unnecessarily:

$$E_m^3 = c_2 e_1 E\left[(N-D) I_{[Y_1>0]} \right].$$

So the expected cost (E_m) is expressed by $E_m = E_m^1 + E_m^2 + E_m^3$. Specifically, it is:

$$\begin{aligned} E_m &= c_0 m + c_0 (N-m) P(Y_1>0) + c_1 E\left[I_{[Y_1=0]} D + e_2 I_{[Y_1>0]} D \right] + c_2 e_1 E\left[(N-D) I_{[Y_1>0]} \right] \\ &= c_0 m + c_0 (N-m) P(Y_1>0) + c_1 E\left[(1-I_{[Y_1>0]}) D + e_2 I_{[Y_1>0]} D \right] + c_2 e_1 E\left[(N-D) I_{[Y_1>0]} \right] \\ &= c_0 m + \left[c_0 (N-m) + c_2 e_1 N \right] P(Y_1>0) + c_1 E[D] - \left[c_1 (1-e_2) + c_2 e_1 \right] E\left[I_{[Y_1>0]} (D_1 + D_2) \right] \end{aligned}$$

(3.1)

where: $P(Y_1>0) = 1 - P(Y_1=0) = 1 - \left\{ \pi E\left[E\left[P(Y_1=0 \mid D_1) \mid D \right] \right] + (1-e_1)^m (1-\pi) \right\}$;

$$E\left[E\left[P(Y_1=0 \mid D_1) \mid D \right] \right] = \sum_{D=0}^{N} \sum_{D_1=0}^{\min(m,D)} \frac{\binom{D}{D_1}\binom{N-D}{m-D_1}}{\binom{N}{m}} \binom{N}{D} p^D (1-p)^{N-D} (1-e_1)^{m-D_1} e_2^{D_1};$$

$E\left[E\left[P(Y_1=0 \mid D_1) \mid D \right] \right] = 1 - E\left[E\left[P(Y_1>0 \mid D_1) \mid D \right] \right]$;

$E[D] = \pi N p$;

$E\left[I_{[Y_1>0]} (D_1 + D_2) \right] = \pi \left\{ E\left[E\left[D_1 P(Y_1>0 \mid D_1) \mid D \right] \right] + E[D_2] E\left[E\left[P(Y_1>0 \mid D_1) \mid D \right] \right] \right\}$;

$E[D_2] = \pi (N-m) p$;

$$E\left[E\left[D_1 P(Y_1>0 \mid D_1) \mid D \right] \right] = \sum_{D=0}^{N} \sum_{D_1=0}^{\min(m,D)} \frac{\binom{D}{D_1}\binom{N-D}{m-D_1}}{\binom{N}{m}} \binom{N}{D} p^D (1-p)^{N-D} \left[1 - (1-e_1)^{m-D_1} e_2^{D_1} \right] D_1$$

and $E(\bullet \mid \bullet) \rightarrow$ denotes the conditioned expectation value.

4. Markov chains in zero-defect acceptance sampling with rectification in a presence of diagnosis errors.

The process of sampling and inspection and the decision to accept or not the lot after the inspection presented in Section 3 can be modeled as a non-irreducible Markov chain with transition matrix $\boldsymbol{P}$. The set of states can be denoted by the vector (s, j, k, t, z), such that $s + j + k + t = z$ and $z = 0, \ldots, m$, being absorbent or transitory states. The variable s can be viewed as the number of conforming items correctly classified as conforming; j is the number of conforming items incorrectly judged as non-conforming; k is the number of non-conforming items classified as conforming; t is the number of non-conforming items correctly judged as non-conforming and z the number of items evaluated. The absorbent states $(j = t = 0$ *and* $z=m$ or $j = 1$ *or* $t=1)$ indicate that the inspection procedure of the m items is finished and the lot was accepted or rejected even before finishing the m inspections.

Consider the probabilities in the transition matrix ***P*** conditioned on the random variable D. Let $\boldsymbol{P_1}$ be the transition matrix conditioned on D when it follows a binomial distribution with parameters (N, p) and $\boldsymbol{P_2}$ the transition matrix when D=0. The matrix $\boldsymbol{P_1}$ occurs with probability π and the matrix $\boldsymbol{P_2}$ with probability $(1-\pi)$.

The probabilities in the transition matrices $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$, related to the inspected lot are respectively:

$$\boldsymbol{P}_1 \rightarrow \begin{cases} P\left[(s+1, j,k,t,z+1) \mid (s, j,k,t,z)\right] = A_1 \\ P\left[(s, j+1,k,t,z+1) \mid (s, j,k,t,z)\right] = A_2 \\ P\left[(s, j,k+1,t,z+1) \mid (s, j,k,t,z)\right] = A_3 \\ P\left[(s, j,k,t+1,z+1) \mid (s, j,k,t,z)\right] = A_4 \\ P\left[(s, j,k,t,z) \mid (s, j,k,t,z)\right] = 1,\ if\ \ z = m\ or\ j = 1\ or\ t = 1 \\ Otherwise\ \ = 0 \end{cases}$$

$$\boldsymbol{P}_2 \rightarrow \begin{cases} P\left[(s+1, j,0,0,z+1) \mid (s, j,0,0,z)\right] = (1-e_1) \\ P\left[(s, j+1,0,0,z+1) \mid (s, j,0,0,z)\right] = e_1 \\ P\left[(s, j,0,0,z) \mid (s, j,0,0,z)\right] = 1,\ if\ \ z = m\ or\ j = 1 \\ Otherwise\ \ = 0 \end{cases}$$

where $A_1 = (1-e_1)\sum_{D=0}^{N-z} \frac{N-z-D}{N-z} \frac{\binom{N}{D} p^D (1-p)^{N-D}}{\sum_{D=0}^{N-z}\binom{N}{D} p^D (1-p)^{N-D}}$;

$$A_2 = e_1 \sum_{D=0}^{N-z} \frac{N-z-D}{N-z} \frac{\binom{N}{D} p^D (1-p)^{N-D}}{\sum_{D=0}^{N-z}\binom{N}{D} p^D (1-p)^{N-D}};$$

$$A_3 = e_2 \sum_{D=0}^{N-z} \frac{D}{N-z} \frac{\binom{N}{D} p^D (1-p)^{N-D}}{\sum_{D=0}^{N-z}\binom{N}{D} p^D (1-p)^{N-D}};$$

and

$$A_4 = (1-e_2)\sum_{D=0}^{N-z} \frac{D}{N-z} \frac{\binom{N}{D} p^D (1-p)^{N-D}}{\sum_{D=0}^{N-z} \binom{N}{D} p^D (1-p)^{N-D}}.$$

The probability of each state after *m* inspections is given by the row vectors $\boldsymbol{P_1}^{(m)}$ and $\boldsymbol{P_2}^{(m)}$. They are respectively by $\boldsymbol{P_1}^{(m)} = \boldsymbol{P}^0 \boldsymbol{P_1}^m$; $\boldsymbol{P_2}^{(m)} = \boldsymbol{P}^0 \boldsymbol{P_2}^m$ and $\boldsymbol{P}^0 = [1,0,0......,0]$ is the probability of initial state vector. Each element of $\boldsymbol{P}^0, \boldsymbol{P_1}^{(m)}$ and $\boldsymbol{P_2}^{(m)}$ is associated to one state (s, j, k, t, z). In $\boldsymbol{P}^0$, the initial probability of the state $(s = 0, j = 0, k = 0, t = 0, z = 0)$ is equal to one and for other states the probability is equal to zero. In the row vectors $\boldsymbol{P_1}^{(m)}$ and $\boldsymbol{P_2}^{(m)}$, the non-null probabilities indicate the absorbent states deciding by the acceptance or rejection of the lot.

Making $\boldsymbol{P}^{(m)} = \pi \boldsymbol{P_1}^{(m)} + (1\text{-}\pi)\boldsymbol{P_2}^{(m)}$ allows us to calculate easily the probabilities of interest mainly the conditioned one related to the acceptance of the lot. For example, summing up the probabilities of the absorbent states (s, j, k, t, z) such that $j = 0,\ t = 0$ and $z = m$ will provide us the probability of accepting a lot [This is denoted by $P(Y_1 = 0)$]. The probability of non-acceptance of a lot is given by the sum of the probabilities of the absorbent states (s, j, k, t, z), such that $j = 1$ or $t = 1$ in $\boldsymbol{P}^{(m)}$. States with $k > 0$ will indicate the wrong acceptance of the lot and states with $j = 1$ indicates that the lot was rejected wrongly.

5. Determination of the optimum sample size m°

The optimum value of *m* (m°) is one that minimizes (3.1) and it can be obtained by direct search substituting values of $m = 0,....,N$ in (3.1). As *N* is usually a large number, a direct search can be a hard task which may spend much time. We propose a limit $L_1 \le N$ in order to speed up our search. Either accepting or rejecting the lot, the cost to inspect *m* items will be at least $c_0 m$. For the optimum value m°, this will be $c_0 m^\circ$. However, if the inspection is not performed, that is, when $m = 0$, the expected cost will be $E_{m=0} = Np\pi c_1$. As $m^\circ c_0 < E_{m^\circ} \le E_{m=0}$ then it follows $m^\circ \le Np\pi c_1 / c_0$. So a direct search to find m° must be proceeded for all integer values of *m* such that $m \le L_1 = \min\{N; Np\pi c_1 / c_0\}$.

It is known that under some regularity conditions a hypergeometric distribution can be approximated by a binomial distribution and this approximation simplifies the mathematical modeling. In this case, we may obtain a new expression for E_m, denoted by E_m^Δ, when this approximation is considered for the random variable D_1. In this sense, a boundary built for E_m^Δ can also be employed to search the optimum sample size. To find a boundary for E_m^Δ is not an easy task. In order to simplify it, a conditioned boundary on $p_h = D / N$ for E_m^Δ is proposed and then using this result, a new one is proposed for E_m.

Let E_m^* be the cost function of E_m^{Δ} when $p_h = D/N$; $m^{\bullet}$ is its optimum sample size. Examining $\Delta E^*_{m^{\bullet}} = E^*_{m^{\bullet}} - E^*_{m^{\bullet}-1} \le 0$ we obtain, after some algebraic manipulation, the inequality

$$a - \pi b^{m^{\bullet}-1} l - (n - m^{\bullet}) b^{m^{\bullet}-1} k \le 0 \tag{5.1}$$

with $a = (c_0 + nc_2 e_1^2)(1-\pi)$; $b = p_h e_1/(1-e_1) + (1-p_h)$;

$$l = [p_h c_1 (1-e_2) + p_h c_2 e_1][(ne_2 - 1)(e_1 + p_h(1-e_1-e_2) + (1-e_2)]/[1 - e_1 - p_h(1-e_1-e_2)] - [c_0 + nc_2 e_1(e_1 + p_h(1-e_1-e_2))];$$

and

$$k = [\pi p_h c_1 (1-e_2) + \pi p_h c_2 e_1][(1-p_h)(1-e_1-e_2)(e_1 + p_h(1-e_1-e_2))]/[1 - e_1 - p_h(1-e_1-e_2)] - [\pi c_0 (e_1 + p_h(1-e_1-e_2))]$$

A set of inequalities expressed in (5.2) can be obtained from (5.1) as functions of k and l.

$$\begin{cases} a - \pi b^{m^{\bullet}-1} l - (n - m^{\bullet}) b^{m^{\bullet}-1} k \ge 0, & \text{if } l \le 0 \text{ and } k \le 0 \\ a - nb^{m^{\bullet}-1} k \le 0, & \text{if } l<0 \text{ and } k>0 \\ a - \pi b^{m^{\bullet}-1} l \le 0, & \text{if } l>0 \text{ and } k<0 \\ a - \pi b^{m^{\bullet}-1} l - nb^{m^{\bullet}-1} k \le 0, & \text{if } l>0 \text{ and } k>0 \end{cases} \tag{5.2}$$

From (5.2), a boundary L_2 for $m^{\bullet}$ can be proposed in (5.3)

$$L_2 \to \begin{cases} m^{\bullet} = 0, & \text{if } l \le 0 \text{ and } k \le 0 \\ m^{\bullet} \le 1 + \dfrac{\log \frac{a}{nk}}{\log b}, & \text{if } l \le 0 \text{ and } k>0 \\ m^{\bullet} \le 1 + \dfrac{\log \frac{a}{\pi l}}{\log b}, & \text{if } l>0 \text{ and } k \le 0 \\ m^{\bullet} \le 1 + \dfrac{\log \frac{a}{\pi l + nk}}{\log b}, & \text{if } l>0 \text{ and } k>0 \end{cases} \tag{5.3}$$

Negative values in (5.3) indicate that $m^{\bullet} = 0$. The expression (5.3) indicates that values lower than the boundary will result $\Delta E^*_{m^{\bullet}} \le 0$. This implies that for values lower than the boundary exists one and only one minimum value for E_m^*.This value is also the global minimum. However, the boundary expressed in (5.3) is conditioned on the value of p_h and valid for E_m^*.

If we unconditioned it, analyzing all possible values of p_h, we can propose a new boundary for E_m^{Δ} expressed as $L_3 = \max\{L_2(p_h)\}$, $p_h = D/N$, $D=1,...,N$. For values lower than L_3, we have $\Delta E^{\Delta}_{m^{\Delta}} = E^{*}_{m^{\Delta}} - E^{*}_{m^{\Delta}-1} \leq 0$, where m^{Δ} denotes the optimum value. This meant that lower than this boundary, it exists one and only one extreme value for E_m^{Δ}.This is also the global minimum. Moreover, it is known that if $m \leq 0.1N$ [Johnson, 1994], the approximation of a hypergeometric distribution by a binomial distribution can proceed. So, this additional condition must also be verified, that is, if $L_3 \leq 0.1N$. In this case, the boundary L_3 can be employed to delimit a minimum for E_m which will be m^{Δ}.

A strategy to perform a computational search the optimum value m (m°) for the expression (3.1) can be drawn . If $L_1 \leq 0.1N$, search for all values lower than $\min\{L_1; L_3\}$, until finding the minimum value m°. If $L_1 > 0.1N$, search for all values lower than $\min\{0.1N; L_3\}$ until finding the minimum value. Compare this result with the search in values higher than $0.1N$ but lower or equal to L_1. The value of m° is the lowest one. Note that the computational search is performed searching only integer values, starting always with the lower one. The flowchart in Figure 3 illustrates the decision process described in this section.

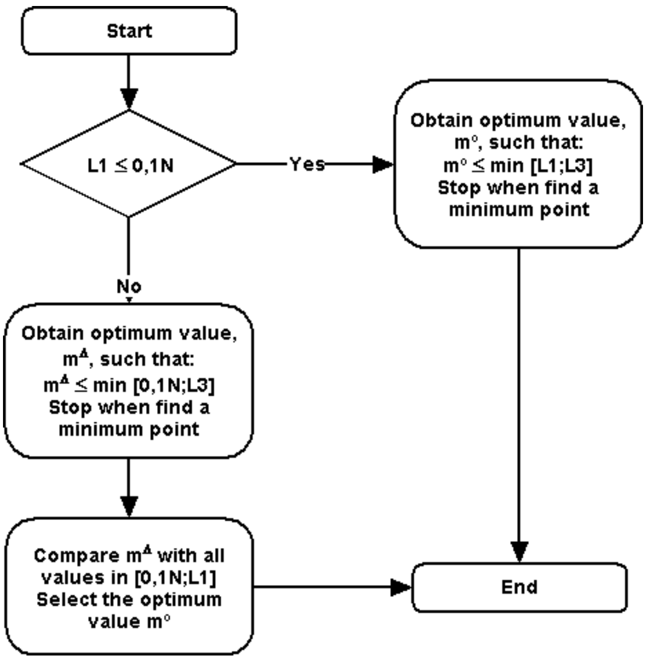
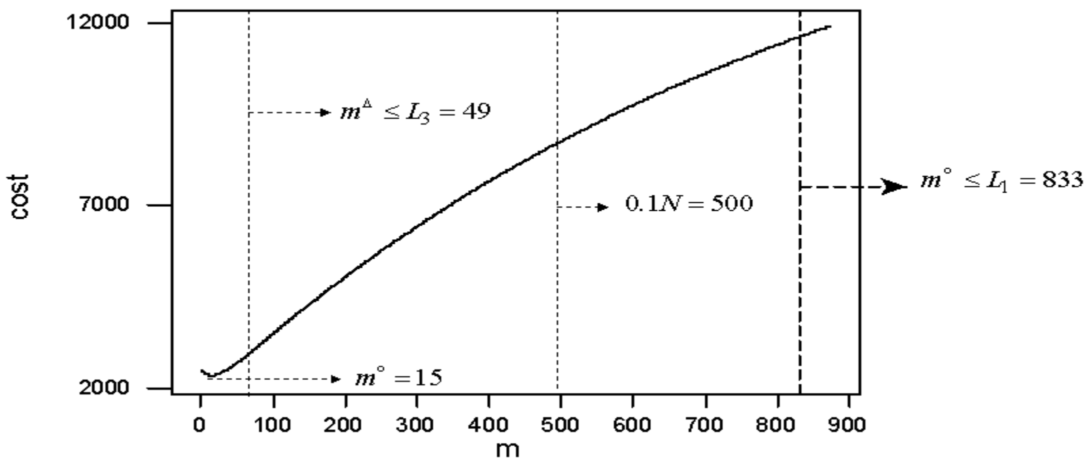


Figure 3: The decision process of the optimum value

6. Numerical Example and Discussions

The example described in this section is based on Hahn (1986), Greenberg (1992), Greenberg (1995) and Anderson (2001). Consider lots with N=5000 items that will be inspected by a zero-defect with rectification procedure. In this context, the following costs are considered: $c_0 = \$3.00$, $c_1 = \$100.00$, $c_2 = \$500.00$, $\pi = 0.1$, $p = 0.05$, $e_1 = e_2 = 0.001$.



behavior of E_m as function of the value of m.

Figure 4: Values of m versus expected cost

A sensitivity analysis was performed to evaluate the behavior of the optimum values of *m* as functions of the parameters. Since all possible scenarios can result in a high number of possibilities to examine, and analyzing all of them can become unmanageable, here we choose to analyze the behavior varying one parameter at a time. The ranges of the parameters explored in this analysis are

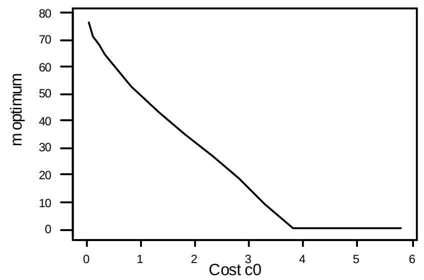
- $0 \le c_0 \le 5$;
- $0 \le c_1 \le 700$;
- $0 \le c_2 \le 2900$;
- $0 \le e_1 \le 0.002$;
- $0 \le e_2 \le 0.15$ and
- $0 \le p \le 1$.

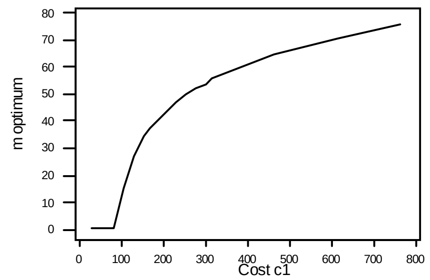
The results of this analysis are plotted in Figure. 5. We observed that as c_0 increases m° tends to zero, indicating the best option is a non-sampling procedure. In the absence of the diagnosis errors, as c_0 tends to zero, the optimum value m° increases to a value higher than *N=5000*. This fact points out the strong influence of the diagnosis errors in obtaining the optimum value m°.

If c_2 increases, the value of m° tends to zero, which justifies when we observe $m^{\circ} = 0$ the possibility of a cost c_2 is eliminated. If c_1 increases the value of m° tends to *N*. This indicates that the proposed procedure is economically feasible.

If e_1 and e_2 increase, it is not feasible making sampling with rectification since the amount of items wrongly classified will remove the benefit of the proposed procedure which is to provide us an accepted lot with lower amount of non-conforming items.

If $p \downarrow 0$, $m^{\circ} \downarrow 0$ since there is only conforming items in the lot. As $p \uparrow 1$, $m^{\circ} \downarrow 1$. This can be justified since the probability to reject the lot alters slightly when $m > 1$ indicating that there is no necessity to sample more than one item.





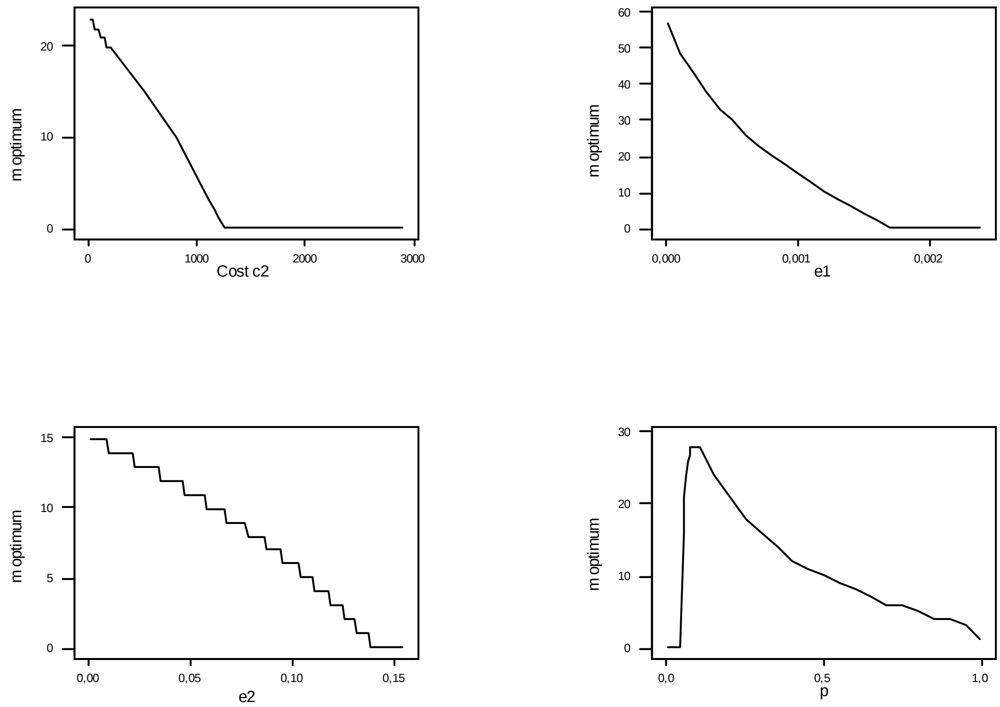


Figure. 5. Optimum values of m versus costs c_0, c_1, c_2, p, e_1, and e_2

Now using Markov chain approach and considering a sample size of 15 items (m°), the probability to accept the lot is 0.9323 and to reject the lot is 0.0677. Possible routes to accept and to reject the lot are listed respectively in Tables 1 and 2. The vectors described in these tables are elements of $\boldsymbol{P}^{(m)}$. Such vector allows us to verify which absorbent states indicate the corrected acceptance of the lot or the corrected rejection of the lot. With the vector $\boldsymbol{P}^{(m)}$ it is possible to calculate conditional probabilities of interest. If the lot was accepted, the probability to be correctly accepted is 0.999961. Similarly, if the lot was rejected, the probability to be correctly rejected is 0.787101. These results suggest a tighter verification in the rejected lots as an attempt to decrease the number of lots rejected wrongly.

s	**j**	**k**	**t**	**Z**	**Probability**
15	0	0	0	15	0.932233125000000000000000000000
14	0	1	0	15	0.000036066879999999997000000000
13	0	2	0	15	0.000000013301099000000003000000
12	0	3	0	15	0.000000000003036620600000000400
11	0	4	0	15	0.000000000000000479946350000000
10	0	5	0	15	0.000000000000000000055628364000
9	0	6	0	15	0.000000000000000000000004884566
8	0	7	0	15	0.000000000000000000000000000331
Sum of the values					0.932269205184136100598986249297

Table 1: Absorbent states – lot accepted

s	J	K	t	z	**Probability**
0	0	0	1	1	0.004995000000000002000000000000
1	0	0	1	2	0.004740504700000001100000000000
2	0	0	1	3	0.004498976000000000900000000000
3	0	0	1	4	0.004269753200000001200000000000
4	0	0	1	5	0.004052209299999999900000000000
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
6	0	7	1	14	0.000000000000000000000000000005
5	0	7	1	13	0.000000000000000000000000000002
0	1	6	0	7	0.000000000000000000000000000001
4	0	7	1	12	0.000000000000000000000000000001
Sum of the probabilities					0,067731000000000005100000000009

Table 2: Absorbent states – lot rejected

7. Conclusions and final remarks

Diagnosis errors can cause a significant impact in determining the optimum sample size in a zero-defect with rectification procedure. As illustrated in this study, even small diagnosis errors as $e_1 = 0.001$ and $e_2 = 0.001$, they can alter significantly the value of optimum *m* (m°). In this sense, it is fundamental to consider the diagnosis errors. They must be incorporated in the model and evaluated in an economic perspective.

Extensions of this study can be made in two directions. One is to change the initial criteria in the sampling inspection for a value other than zero, that is $c, c \geq 0$. Another alternative is to make repetitive tests to minimize the effect of the diagnosis errors. An item would be classified as conforming if the number of conforming independent classifications is higher than a specified value *a*. In this scenery, the objective is to determine the optimum values of *m*, the number of the independent repetitive inspections in an item, the value of *a* and the value of *c* such that minimize the total expected cost.

References

(1) Aminzadeh, M. S. (2003). Bayesian Economic Variable Acceptance-Sampling Plan Using Inverse Gaussian Model and Step-Loss Function, Communication in Statistics – Theory and Methods, **32**(5), 961-982.

(2) Anderson M. A.; Greenberg B. S. and Stokes S. L. (2001). Acceptance sampling with rectification when inspection errors are present, Journal of Quality Technology, **33**(4), 493-505.

(3) Brush G. G.; Hoadley B., and Saperstein B. (1990). Estimating outgoing quality using the quality measurement plan. Technometrics, **32**, 31-41.

(4) Ferrell G.W. and Chhoker A. (2002). Design of economically optimal acceptance sampling plans with inspection error, Computers & Operations Research, **29**, 1283-1300.

(5) Greenberg B. S. and Stokes S. L. (1992). Estimating nonconformance rates after zero defect sampling with rectification. Technometrics, **34**(2), 203-213.

(6) Greenberg B.S. and Stokes S.L. (1995). Repetitive testing in the presence of inspection errors. Technometrics, **37**(1), 102-111.

(7) Hahn G.J. (1986). Estimating the percent nonconforming in the accepted product after zero defect sampling, Journal of Quality Technology, **18**(3), 182-188.

(8) Johnson N.L.; Kotz S. and Balakrishnan N. (1992). Univariate Discrete Distributions, Second Edition, John Wiley & Sons, Inc., New York.

(9) Johnson N.L.; Kotz S. and Wu X. (1991). Inspection errors for attributes in quality control, Chapman & Hall, London.

(10) Hald, A. (1981). Statistical Theory of Sampling Inspection by Attributes. Academic Press, New York.

(11) Markowski E.P. and Markowski C.A. (2002). Improved attribute acceptance sampling plans in the presence of misclassification error, European Journal of Operation Research, **139**(3), 501-510.

(12) Minton G. (1972). Verification error in single sampling inspection plans for processing survey data, Journal of the American Statistical Association, **67**, 46-54.

(13) Quinino R.C. and Ho L.L. (2003). Repetitive tests as an economic alternative procedure to control attributes with diagnosis errors, European Journal of Operation Research, Article in Press.

(14) Quinino R.C. and Suyama,E. (2002). Número ótimo de classificações independentes com erro na avaliação da conformidade de produtos, Pesquisa Operacional, **22**(1), 1-8.

(15) Starbird, A. (1997). Acceptance sampling, imperfect production, and the optimality of zero defects. Naval Research Logistics, **44**, 515-530.

(16) Wetherrill G.B. and Chiu W. K. (1975). A review of acceptance sampling schemes with emphasis on the economic aspect, International Statistical Review, **43**(2), 91-210.

Appendix

Use a Matlab Editor to write the files *.m. Run optimum.m the software Matlab.

```
% optimum.m
clear all;
global c0 c1 c2 pi N e1 e2
tic;
c0 = 3;
c1 = 100;
c2 = 500;
pi = 0.1;
p = 0.05;
N = 650;
e1 = 0.001;
e2 = 0.001;
L1=ceil(min(N,N*p*pi*c1/c0));
L2=ceil(0.1*N);
L3=ceil(limiteL3(N));
if L2 > L1
   achouparou=1;
   faixa1=[1:1:min(L1,L3)];
   faixa2=[];
elseif L2 < L3
   achouparou=0;
   faixa1=[1:1:L2];
   faixa2=[L2:1:L1];
else
   achouparou=0;
   faixa1=[1:1:L3];
   faixa2=[L2:1:L1];
end
s5=inf;
s4=1e30;
i=0
while i<length(faixa1) & s4<s5
   i=i+1;
   s5=s4;
   m=faixa1(i);
   progresso(m,faixa1,faixa2);
   s1=0; s2=0; s3=0;

tbinom=binopdf(0:1:N,linspace(N,N,N+1),linspace(p,p,
N+1));
   for D=0:N
      minimo=min(m,D);
      D1=0:1:minimo;

thiper=tbinom(D+1)*hygepdf(D1,linspace(N,N,minimo
+1),

linspace(D,D,minimo+1),linspace(m,m,minimo+1));
      s1=s1+sum(thiper.*(1-(1-e1).^(m-
D1).*e2.^D1).*D1);
      s2=s2+sum(thiper.*(1-(1-e1).^(m-D1).*e2.^D1));
      s3=s3+sum(thiper.*(1-e1).^(m-D1).*e2.^D1);
   end
   Um = 1-(pi*s3+(1-e1)^m*(1-pi));
   custo(i) = c0*m + c0*(N-m)*Um + c1*pi*N*p -
c1*(1-e2)*pi*s1
    - c1*(1-e2)*pi*(N-m)*p*s2 + c2*N*e1*Um -
c2*e1*pi*s1 - c2*e1
    *pi*(N-m)*p*s2;
   amostra(i)=m;
   s4=custo(i);
end
if achouparou==0
   for j=1:length(faixa2)
      m=faixa2(j);
      progresso(m,faixa1,faixa2);
      s1=0; s2=0; s3=0;

tbinom=binopdf(0:1:N,linspace(N,N,N+1),linspace(p,p,
N+1));
      for D=0:N
         minimo=min(m,D);
         D1=0:1:minimo;

thiper=tbinom(D+1)*hygepdf(D1,linspace(N,N,minimo
+1),

linspace(D,D,minimo+1),linspace(m,m,minimo+1));
         s1=s1+sum(thiper.*(1-(1-e1).^(m-
D1).*e2.^D1).*D1);
         s2=s2+sum(thiper.*(1-(1-e1).^(m-
D1).*e2.^D1));
         s3=s3+sum(thiper.*(1-e1).^(m-D1).*e2.^D1);
      end
      Um = 1-(pi*s3+(1-e1)^m*(1-pi));
      custo(i+j) = c0*m + c0*(N-m)*Um + c1*pi*N*p -
c1*(1-e2)*pi*s1 –
      c1*(1-e2)*pi*(N-m)*p*s2 + c2*N*e1*Um -
c2*e1*pi*s1 -
      c2*e1*pi*(N-m)*p*s2;
      amostra(i+j)=m;
   end
end
amostra=[0 amostra]
custo=[N*p*pi*c1 custo]
[Minimo, pos]=min(custo);
Optimum=amostra(pos);
clc;
fprintf('%60s\n','*******************************
*******************************************
*********');
fprintf('%60s\n','                              Result');
fprintf('%50s\n','*******************************
*******************************************
**********');
fprintf('%2s\n',' ');
fprintf('%40s\t %10.6f\n','Expected Cost              =
',Minimo);
fprintf('%40s\t %6.0f\n',' m optimum                  =
',Optimum);
fprintf('%2s\n',' ');
fprintf('%40s\t %4.6f\n',' Time (min)                 =
',toc/60);
fprintf('%2s\n',' ');
fprintf('%50s\n','*******************************
*******************************************
*********');
```

```
%limitel3.m
function y=limiteL3(intervalo)
global c0 c1 c2 pi N e1 e2
for z=0:intervalo
    w=z/intervalo;
    p = w;
    a=(c0+N*c2*e1*e1)*(1-pi);
    b=(p*e2/(1-e1)+(1-p));
    c=(c1*(1-e2)+c2*e1)*p;
    d=(N*e2-1)*(e1+p*(1-e1-e2))+1-e2;
    e=1-e1-p*(1-e1-e2);
    f=(c0+N*c2*e1*(e1+p*(1-e1-e2)));
    g=c0*(1-pi)*e1;
    h=(1-e1-e2)*(1-p);
    i=(e1+p*(1-e1-e2));
    k=pi*c*h*i/e-c0*pi*i;
    l=c*d/e-f;
    m1=0;
    m2=a/(N*k);
    m3=a/(pi*l);
    m4=a/(pi*l+N*k);
    if l<=0 & k<=0
        L1=m1;
        mm(z+1)=L1;
    elseif l<=0 & k>0
        L2=1+(log(m2)/log(b));
        mm(z+1)=L2;
    elseif l>0 & k<=0
        L3=1+(log(m3)/log(b));
        mm(z+1)=L3;
    elseif l>0 & k>0
        L4=1+(log(m4)/log(b));
        mm(z+1)=L4;
    end
    pp(z+1)=p;
end

L=max(mm)
L=floor(L)
y=L;
```

```
%progresso.m
function y=progresso(m,faixa1,faixa2)
clc;
min1=min(faixa1);
max1=max(faixa1);
min2=min(faixa2);
max2=max(faixa2);

fprintf('%60s\n','******************************
*******************************************
********');
fprintf('%60s\n','                                   Progress');
fprintf('%50s\n','******************************
*******************************************
**********');
fprintf('%2s\n','  ');
fprintf('%40s\t %6.0f\n','Boundary 1 (min)              =
',min1);
fprintf('%40s\t %6.0f\n','Boundary 1 (max)              =
',max1);
fprintf('%2s\n','  ');
fprintf('%40s\t %6.0f\n','Boundary 2 (min)              =
',min2);
fprintf('%40s\t %6.0f\n','Boundary 2 (max)              =
',max2);
fprintf('%2s\n','  ');
fprintf('%40s\t %6.0f\n','m                             = ',m);
fprintf('%2s\n','  ');
fprintf('%40s\t %4.6f\n','time (min)                    =
',toc/60);
fprintf('%50s\n','******************************
*******************************************
**********');
```