Soluções da lista 2.1 sobre TCL (Sheldom Ross)

1) (8.2 c), (8.3)

$$P\left\{\left|\sum_{i=1}^{n} X_i/n - 75\right| > 5\right\} \approx P\{|Z| > \sqrt{n}\} \leq .1 \text{ when } n = 3$$

2) (8.4)

$$P\left\{\sum_{i=1}^{20} X_i > 15\right\} = P\left\{\sum_{i=1}^{20} X_i > 15.5\right\}$$
$$\approx P\left\{Z > \frac{15.5-20}{\sqrt{20}}\right\}$$
$$= P\{Z > -1.006\}$$
$$\approx .8428$$

3) (8.5)

Letting X_i denote the i^{th} roundoff error it follows that $E\left[\sum_{i=1}^{50} X_i\right] = 0$,

$\text{Var}\left(\sum_{i=1}^{50} X_i\right) = 50\ \text{Var}(X_1) = 50/12$, where the last equality uses that $.5 + X$ is uniform (0, 1) and so $\text{Var}(X) = \text{Var}(.5 + X) = 1/12$. Hence,

$$P\left\{\left|\sum X_i\right| > 3\right\} \approx P\{|N(0,1)| > 3(12/50)^{1/2}\} \text{ by the central limit theorem}$$
$$= 2P\{N(0,1) > 1.47 = .1416$$

4) (8.6)

If X_i is the outcome of the i^{th} roll then $E[X_i] = 7/2 \quad \text{Var}(X_i) = 35/12$ and so

$$P\left\{\sum_{i=1}^{79} X_i \leq 300\right\} = P\left\{\sum_{i=1}^{79} X_i \leq 300.5\right\}$$
$$\approx P\left\{N(0,1) \leq \frac{300.5 - 79(7/2)}{(79 \times 35/12)^{1/2}}\right\} = P\{N(0,1) \leq 1.58\} = .9429$$

5) (8.7)

$$P\left\{\sum_{i=1}^{100} X_i > 525\right\} \approx P\left\{N(0,1) > \frac{525-500}{\sqrt{(100\times 25)}}\right\} = P\{N(0,1) > .5\} = .3085$$

where the above uses that an exponential with mean 5 has variance 25.

6) (8.10)

If W_n is the total weight of n cars and A is the amount of weight that the bridge can withstand then $W_n - A$ is normal with mean $3n - 400$ and variance $.09n + 1600$. Hence, the probability of structural damage is

$$P\{W_n - A \geq 0\} \approx P\left\{Z \geq (400-3n)/\sqrt{.09n+1600}\right\}$$

Since $P\{Z \geq 1.28\} = .1$ the probability of damage will exceed .1 when n is such that

$$400 - 3n \leq 1.28\sqrt{.09n+1600}$$

The above will be satisfied whenever $n \geq 117$.