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Invited Review

Imperfect maintenance

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Abstract

The maintenance of a deteriorating system is often imperfect: the system after maintenance will not as good as new, but younger. Imperfect maintenance study indicates a significant breakthrough in reliability and maintenance theory. Research activities in maintenance engineering have been conducted over the past 30 years and more than 40 mathematical imperfect maintenance models have been proposed for estimating the reliability measures and determining the optimum maintenance policies. Various treatment methods and optimal policies on the imperfect maintenance are discussed and summarized in this paper. Several important results for imperfect maintenance are also presented.

Keywords: Imperfect maintenance; Reliability; Optimal maintenance policy; Minimal repair; Preventive maintenance; Renewal theory

1. Introduction

Maintenance involves preventive (planned) and unplanned actions carried out to retain a system in or restore it to an acceptable operating condition. Optimal maintenance policies aim to provide optimum system reliability/availability and safety performance at lowest possible maintenance costs. Proper maintenance techniques have been emphasized in recent years due to increased safety and reliability requirements of systems, increased complexity, and rising costs of material and labor (Sherif and Smith, 1981). For some critical systems, for example, in air traffic control, aircraft, realtime military applications, automotive mechanical and safety control, and hospital patient monitoring systems, it is extremely important to avoid failure during actual operation because it is dangerous and disastrous (Pham, 1995). One important research area in reliability engineering is the study of various maintenance policies in order to prevent the occurrence of system failure and improve system availability.

In the past several decades, maintenance, replacement and inspection problems have been extensively discussed in the literature. Maintenance can be classified by two major categories: corrective or preventive. Corrective maintenance (CM) is any maintenance that occurs when the system is failed. Some authors refer to corrective maintenance as repair and we will use them interchangeably throughout this paper. According to MIL-STD-721B, corrective maintenance means all actions performed as a result of failure, to restore an item to a specified condition. Preventive maintenance (PM) is any maintenance that occurs when the system is operating. According to MIL-STD-721B, preventive maintenance means all actions performed in an attempt to retain an item in specified condition by providing systematic in-

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spection, detection, and prevention of incipient failures. We think that maintenance can be classified according to the *degree* to which the operating conditions of an item is restored by maintenance in the following way:

a) Perfect repair or perfect maintenance: a maintenance action which restores the system operating condition to as good as new. That is, upon perfect maintenance, a system has the same lifetime distribution and failure rate function as a brand new one. Complete overhaul of an engine with a broken connecting rod is an example of perfect repair. Generally, replacement of a failed system by a brand new one is a perfect repair.

b) Minimal repair or minimal maintenance: a maintenance action which restores the system to the failure rate it had when it failed. Minimal repair is first studied by Barlow and Proshan (1965) and after it the system operating state is often called as bad as old. Changing a flat tire on a car or changing a broken fan belt on an engine are examples of minimal repair because the overall failure rate of the car is essentially unchanged.

c) Imperfect repair or imperfect maintenance: a maintenance action does not make a system like as good as new, but younger. Usually, it is assumed that imperfect maintenance restores the system operating state to somewhere between as good as new and as bad as old. Clearly, imperfect repair (maintenance) is a general repair (maintenance) which can include two extreme cases: minimal and perfect repair (maintenance). Engine tune-up is an example of imperfect maintenance because an engine tune-up may not make an engine as good as new but its performance might be greatly improved.

d) Worse repair or maintenance: a maintenance action which makes the system failure rate or actual age increases but the system does not break down. Thus, upon worse repair system's operating condition becomes worse than that just prior to its failure.

e) Worst repair or maintenance: a maintenance action which undeliberately makes the system fail or break down.

Some possible causes for imperfect, worse or worst maintenance due to the maintenance performer, for example (Brown and Proschan, 1983):

- Repair the wrong part
- Only partially repair the faulty part
- Repair (partially or completely) the faulty part but damage adjacent parts
- Incorrectly assess the condition of the unit inspected
- Perform the maintenance action not when called for but at his convenience (the timing for maintenance is off the schedule).

Several other reasons causing worse or worst maintenance (Nakagawa and Yasui, 1987):

- Hidden faults and failures which are not detected during maintenance
- Human errors such as wrong adjustments and further damage done during maintenance
- Replacement with faulty parts.

According to Brown and Proschan (1983), maintenance policies based on planned inspections are periodic inspection and inspection interval dependent on age. By periodic inspections, a failed unit is identified (e.g., spare battery, a fire detection device, etc.) or it is determined whether the unit is functioning or not. With ageing of the unit, the inspection interval may be shortened. These inspection methods are subject to imperfect maintenance caused by randomness in actual time of inspection in spite of the schedule, imperfect inspection, and cost structure. Therefore, realistic and valid methods must incorporate random features of the inspection policy.

According to the above classifications, we can say that a preventive maintenance is a minimal, perfect, imperfect, worse or worst preventive maintenance. Similarly, a corrective maintenance may be a minimal, perfect, imperfect, worse or worst corrective maintenance. We will refer to imperfect corrective and preventive maintenance as imperfect maintenance later.

The type and degree of maintenance that is used in practice depend on the types of applications, system costs as well as reliability and safety requirements. Earlier studies and results of preventive maintenance models usually assumed that (1) the system after corrective or preventive maintenance is as good as new (perfect maintenance) or as bad as old (minimal maintenance), and (2) the repair or maintenance times are both assumed to be negligible. The perfect maintenance assumption in deed may be reasonable for systems with only one component which is structurally simple. On the other hand, the minimal repair assumption seems plausible for failure behavior of systems when one of its many, nondominating components is replaced by a new one (Kijima, 1989).

These two assumptions may not be true in many applications, especially in multicomponent systems such as aircraft, fly-by-wire, medical equipment, nuclear reactors, etc. Many maintenance activities may realisticly not result in such two extreme situations but in a complicated intermediate one. That is, when the system is maintained correctively or preventively, its failure rate is somewhere between as good as new and as bad as old. In other words, imperfect maintenance is the concept that maintenance actions do not make the system to as good as new condition but rather bring the state of a failed system to a level which is somewhere between new and prior to failure. This is called imperfect corrective or preventive maintenance. For example, an engine on a car may not be as good as new or as bad as old, but younger after tune-up. Perfect maintenance and minimal maintenance are often found very limited uses in practical applications, if not at all, therefore realistic imperfect maintenance should be further investigated. Recently, imperfect corrective and preventive maintenance has received lots of attention in reliability and maintenance work. There are a great number of journals where many interesting research papers on imperfect maintenance can be found, for example, Advances in Applied Probability, IEEE Transactions on Reliability, International Journal of Reliability, Quality and Safety Engineering, Journal of Applied Probability, Microelectronics and Reliability, Naval Research Logistics Quarterly, Reliability Engineering and Systems Safety, and this journal, to name a few.

In this paper, we discuss current treatment methods and optimal maintenance policies of single- and multi- component systems the rapidly growing literature on the subject of imperfect maintenance with the purpose to overview the recent maintenance models and policies, methodologies and techniques, tools, and applications to indicate new directions of research and to stimulate further research in the field. Because of a great deal of research has been done and the rapidly growing literature, our discussion in this paper mainly focus on the models, policies, and results that have appeared only after 1985. The reader is referred to Sherif and Smith (1981), Jardine and Buzacott (1985), Valdez-Flores and Feldman (1989), and Cho and Parlar (1991), for the works and results that appeared before 1985.

2. Treatment methods for imperfect maintenance

To our knowledge, Kay (1976) and Chan and Downs (1978) studied the worst PM. Ingle and Siewiorek (1977) investigated imperfect maintenance. Chaudhuri and Sahu (1977) mention the concept of imperfect PM. Most research on imperfect, worse and worst maintenance is for a single-unit system. Some researchers have proposed various methods for modeling imperfect, worse and worst maintenance. It is necessary to summarizing these methods. This will be helpful for later research on imperfect, worse and worst maintenance because these modeling methods can be utilized in various maintenance and inspection policies. Although these methods will be summarized mainly from the work for a single-unit system they will be useful for modeling multicomponent systems because the study methods of imperfect, worse, worst maintenance for single-unit systems will also be effective for modeling the imperfect maintenance of individual subsystems that are part of multicomponent systems.

Methods for treating imperfect, worse and worst maintenance can be classified into eight categories and we now discuss them as follows.

2.1. Treatment method 1 — (p,q) rule

Nakagawa (1979a,b) treats the imperfect PM in this way: the component is returned to the as good as new state (perfect PM) with probability p and it is returned to the as bad as old state (minimal PM) with probability $q = 1 - p$ after PM. Clearly, if $p = 1$ the PM coincides with perfect one and if $p = 0$ it corresponds to minimal PM. In this sense, minimal and perfect maintenance's are special cases of imperfect maintenance and imperfect maintenance is a general maintenance. From this proposed imperfect PM model, Nakagawa (1979a,b, 1980) then succeeded to obtain optimum preventive maintenance policies minimizing the expected maintenance cost rate for one-unit system under age-dependent and periodic PM policies.

Brown and Proschan (1983) consider the following model of the repair process. A unit is repaired each time it fails. The executed repair is either a perfect repair with probability p or a minimal repair with probability $1-p$. Assuming that all repair actions take negligible time, they establish ageing preservation properties of this imperfect repair process and monotonicity of various parameters and random variables associated with the failure process. They obtain an important, useful result: if the life distribution of a unit is F and its failure rate is r, then the distribution function of the time between successive perfect repairs $F_p = 1-(1-F)^p$ and the corresponding failure rate $r_p = pr$. Using this result, Fontenot and Proschan (1984) and Wang and Pham (1996b) obtain the optimal imperfect maintenance policies for one-component system. Later on, we will refer to this method of modeling imperfect maintenance as (p, q) rule, that is, after maintenance (corrective or preventive) a system becomes as good as new with probability p and as bad as old with probability $1-p$. In fact, some other imperfect maintenance models have used this rule in recent studies.

Bhattacharjee (1987) obtains the same results as in Brown and Proschan (1983) using different methods, and some other new results for the Brown–Proschan model of imperfect repair via a shock model representation of the sojourn time.

2.2. Treatment method 2 — $(p(t), q(t))$ rule

Block et al. (1985) extended the above Brown–Proschan imperfect repair model with the (p, q) rule to the age-dependent imperfect repair for one-unit system: An item is repaired at failure (corrective maintenance). With probability $p(t)$, the repair is a perfect repair; and with $q(t) = 1-p(t)$, the repair is a minimal repair, where t is the age of the item in use at the failure time (the time since the last perfect repair). Block et al. (1985) show that if the item's life distribution F is a continuous function and its failure rate is r, the successive perfect repair times is a renewal process with interarrival time distribution

$$F_p(t) = 1 - \exp\left(\int_0^t \frac{p(x)}{1-F(x)}\,\mathrm{d}x\right)$$

and the corresponding failure rate

$$r_p(t) = p(t)r(t)$$

Block, Borges and Savits prove that the ageing preservation results of Brown and Proschan (1983) hold under suitable hypotheses on $p(y)$. Later on we will call this imperfect maintenance modeling method as $(p(t), q(t))$ rule.

Using this $(p(t), q(t))$ rule, Block et al. (1988) discuss a general age-dependent PM policy, where an operating unit is replaced when it reaches age T; if it fails at age $y < T$, it is either replaced by a new unit with probability $p(t)$, or it undergoes minimal repair with probability $q(t)$. The cost of the ith minimal repair is a function, $c_i(y)$, of age and number of repairs. After a perfect maintenance, planned or unplanned (preventive), the procedure is repeated.

Brown and Proschan's model and Block, Borges and Savits's model assume that the repair time is negligible. Iyer (1992) later obtains availability results for imperfect repair using $(p(t), q(t))$ rule considering that the repair time is not negligible. His realistic treatment method will be helpful for later research. Sumita and Shanthikumar (1988) proposed and studied an age-dependent counting process generated from a renewal process and applied that counting process to the age-dependent imperfect repair for one-unit system.

Whitaker and Samaniego (1989) proposed an estimator for the life distribution when the above model by Block et al. (1985) is observed until the time of the mth perfect repair. This estimator was motivated by a nonparametric maximum likelihood approach, and was shown to be a "neighborhood MLE". They derived large-sample results for this estimator. Hollander et al. (1992) take the new approach of using counting process and martingale theory to analyze these models. Their methods yields extensions of Whitaker and Samaniego's results to the whole line and provide a useful framework for further work on the minimal repair model.

The (p, q) rule and $(p(t), q(t))$ rule for imperfect maintenance seem practical and realistic. It makes imperfect maintenance be somewhere between perfect and minimal ones. The *degree* to which the operating conditions of an item is restored by maintenance can be measured by p or $p(t)$. Especially, in the $(p(t), q(t))$ rule, the *degree* to which the operating conditions of an item are restored by maintenance is related to its age t. Thus, the $(p(t), q(t))$ rule seems more realistic but by considering it mathematical modeling of imperfect maintenance will be more complicated. We think that these two rules can be expected to see more often in future research on the imperfect maintenance modeling. Both rules in fact have received much attention and have been used in many imperfect repair models.

Makis and Jardine (1992) recently considered a general treatment method for imperfect maintenance and model imperfect repair at failure in a way that repair returns a system to the as good as new state with probability $p(n, t)$ or to the as bad as old state with probability $q(n, t)$, or with probability $s(n, t) = 1 - p(n, t) - q(n, t)$ the repair is unsuccessful, the system is scrapped and replaced by a new one, where t is the age of the system and n is the number of failures since replacement. We will refer to this treatment method as $(p(n, t), q(n, t), s(n, t))$ rule later.

2.3. Treatment method 3 — improvement factor method

Malik (1979) introduces the concept of improvement factor in the maintenance scheduling problem. He thinks that maintenance changes the system time of the failure rate curve to some newer time but not all the way to zero (not new). This treatment method for imperfect maintenance also makes the failure rate after PM lies between as good as new and as bad as old. The degree of improvement in failure rate is called improvement factor. Malik assumes that since systems need more frequent maintenance with increased age the successive PM intervals are decreasing in order to keep the system failure rate at or below a stated level, and propose an algorithm to determine these successive PM intervals. Lie and Chun (1986) present a general expression to determine these PM intervals. Here, Malik, however, relied on an expert judgment to estimate the improvement factor, while Lie and Chun give a set of curves as a function of maintenance cost and the age of the system for the improvement factor. Suresh and Chaudhuri (1994) regard the starting condition, ending condition, operating condition, and type of maintenance of a system as fuzzy sets. Improvement factor is used to find out the starting condition of the system after maintenance.

Using the improvement factor and assuming finite planning horizon, Jayabalan and Chaudhuri (1992b) introduced a branching algorithm to minimize the average total cost for a maintenance scheduling model with assured reliability and they discussed optimal maintenance policy for a system with increased mean down time and assured failure rate (Jayabalan and Chaudhuri, 1992c). It is worthwhile to note that using fuzzy set theory and improvement factor, Suresh and Chaudhuri (1994) establish a PM scheduling procedure to assure an acceptable reliability level or tolerable failure rate assuming finite planning horizon.

Chan and Shaw (1993) suggest that failure rate is reduced after each PM and this reduction of failure rate depends on the item age and the number of PM's. They propose two types of failure-rate reduction: (1) failure rate with fixed reduction, that is, after each PM, the failure rate is reduced such that all jump downs of the failure rate are the same; and (2) failure rate with proportional reduction, in other words, after PM, the failure rate is reduced such that each jump down is proportional to the current failure rate. They obtain cycle availability for single unit system and discuss the design scheme to maximize the probability of achieving a specified stochastic cycle availability with respect to the duration of the operating interval between PM's.

This kind of study method for imperfect maintenance is in terms of failure rate or other reliability measures and seems useful and practical in engineering which can be used as a general treatment method for imperfect maintenance or even *worse* maintenance. Later on we will call this treatment approach *improvement factor method.*

Besides, Canfield (1986) assumes that PM at time t restores the failure rate function to its shape at $t - \tau$, while the level remains unchanged where τ is less than or equal to the PM intervention interval.

2.4. Treatment method 4 — virtual age method

Kijima et al. (1988) develop an imperfect repair model by using the idea of the virtual age process of a repairable system. If the system has the virtual age $V_{n-1}=y$ immediately after the $(n-1)$th repair, the nth failure time X_n is assumed to have the distribution function

$$\Pr\{X_n \le x \mid V_{n-1}=y\} = \frac{F(x+y)-F(y)}{1-F(y)},$$

where $F(x)$ is the distribution function of the time to failure of a new system. Let a be the degree of the nth repair where $0 \le a \le 1$. They construct such a repair model: the nth repair cannot remove the damage incurred before the $(n-1)$th repair. It reduces the additional age X_n to aX_n. Accordingly, the virtual age after the nth repair becomes

$$V_n = V_{n-1} + aX_n.$$

Obviously, $a=0$ corresponds to a perfect repair while $a=1$ to a minimal repair. Later Kijima (1989) extended the above model to the case that a is a random variable taking a value between 0 and 1 and proposes another imperfect repair model where A_n is a random variable taking a value between 0 and 1 for $n=1,2,3,\ldots$ For the extreme values 0 and 1, $A_n = 1$ means a minimal repair and $A_n = 0$ a perfect repair. Comparing this treatment method with Brown and Proschan's, we can see that if A_n is independently and identically distributed (i.i.d.) taking the two extreme values 0 and 1 then they are the same. Therefore, the second treatment method by Kijima (1989) is general. He derives various monotonicity properties associated with the above two models. This treatment method will be referred to as *virtual age method* later.

It is worth to mention that Uematsu and Nishida (1987) consider a more general model including the above two models by Kijima (1989) as special cases and obtain some elementary properties of the associated failure process. Let T_n denote the time interval between the $(n-1)$st failure and the nth one, and X_n denote the degree of repair. After performing the nth repair, the age of the system is $q(t_1,\ldots,t_n;\, x_1,\ldots,x_n)$ given that $T_i=t_i$ and $X_i=x_i$ $(i=1,2,\ldots,n)$ where T_i and X_i are random variables. On the other hand, $q(t_1,\ldots,t_n;\, x_l,\ldots,x_{n-1})$ represents the age of the system just before the nth failure. The starting epoch of an interval is subject to the influence of all previous failure history, i.e., the nth interval is statistically dependent on $T_1=t_1,\ldots,T_{n-1}=t_{n-1}$, $X_1=x_1,\ldots,X_{n-1}=x_{n-1}$. For example, if

$$q(t_1,\ldots,t_n;\, x_1,\ldots,x_n) = \sum_{j=1}^{n}\sum_{i=j}^{n} x_i t_j,$$

then $X_i=0$ $(X_i=1)$ represents that perfect repair (minimal repair) performs at the ith failure.

2.5. Treatment method 5 — shock model method

It is well known that the time to failure of a unit can be represented as a first passage time to a threshold for an appropriate stochastic process that describes the levels of damage.

Consider a unit which is subject to shocks occurring randomly in time. At time $t=0$, the damage level of the unit is assumed to be 0. Upon occurrence of a shock, the unit suffers a nonnegative random damage. Each damage, at the time of its occurrence, adds to the current damage level of the unit, and between shocks, the damage level stays constant. The unit fails when its accumulated damage first exceeds a specified level. To keep the unit in an acceptable operating condition, some PM is necessary (Kijima and Nakagawa, 1991).

Kijima and Nakagawa (1991) propose a cumulative damage shock model with imperfect periodic PM. The PM is imperfect in the sense that each PM reduces the damage level by $100(1-b)\%$, $0 \le b \le 1$, of total damage. Note that if $b=1$ the PM is minimal, and if $b=0$ the PM coincides with a perfect PM. This research approach is similar to the one in treatment method 1. They derive a sufficient condition for the time to failure to have an increasing failure rate (IFR) distribution and discuss the problem of finding the number of PM's that minimizes the expected maintenance cost rate.

Kijima and Nakagawa (1992) establish another cumulative damage shock model with a sequential PM policy assuming that PM is imperfect. They model imperfect PM in the sense that the amount of damage after the kth PM becomes b_kY_k when it was Y_k before PM, i.e., the kth PM reduces the amount Y_k of damage to b_kY_k where b_k is called improvement factor in Kijima and Nakagawa (1992). They assume that a system is subject to shocks occurring according to a Poisson process, and upon occurrence of shocks, it suffers a nonnegative random damage which is additive. Each shock causes a system failure with probability $p(z)$ when the total damage is z at the shock. In this model, PM is done at fixed intervals x_k for $k = 1, 2, \ldots, N$ because more frequent maintenance is needed with age, and the Nth PM is perfect (in fact, this a sequential PM policy). If the system fails between PM's it undergoes only minimal repair. They derive the expected maintenance cost rate until replacement assuming that $p(z)$ is an exponential function and damages are independently and identically distributed, and discuss the optimal replacement policies.

This study approach for imperfect maintenance will be called *shock model method* or *shock method* later.

2.6. Treatment method 6 — (α, β) rule

Wang and Pham (1996a–d) treat imperfect repair in such a way that after repair the lifetime of a system will be reduced to a fraction α of the one immediately preceding it, where $0 < \alpha < 1$, that is, the lifetime decreases as the number of repairs increases. The interarrival times between successive repairs constitute a "quasi renewal process" (Wang and Pham, 1996c) if repair time is negligible.

Assuming that the pdf of the lifetime of a system which has been subject to the first $(n-1)$ repairs since it is new, X_n, is $f_n(x)$ for $n = 1, 2, \ldots$, Wang and Pham (1996c) studied this quasi renewal process and proved that:

1. If $f_1(x)$ belongs to IFR, DFR, IFRA, DFRA, NBU, then $f_n(x)$ is in the same category for $n = 2, 3, \ldots$
2. The shape parameter of X_n is the same for $n = 1, 2, 3, \ldots$ for a quasi renewal process if X_1 follows the gamma, Weibull or lognormal distribution.

The second result is that after "renewal" the shape parameters of the interarrival time will not change. In reliability theory, the shape parameters of lifetime of a product tend to relate to its failure mechanism. Usually, if it possesses the same failure mechanism then a product will have the same shape parameters of its lifetimes at different operating conditions. Because most maintenances usually do not change the failure mechanism we can expect that the lifetime of a system will have the same shape parameters. Thus, in this sense a quasi renewal process will be suitable to model the imperfect maintenance process.

Wang and Pham (1996a,b,d) further suggest that repair time is nonnegligible (most other imperfect maintenance models, however, assume that repair and PM time are negligible), and upon repair the next repair time will be increased to a multiple β of the one immediately preceding it where $\beta > 1$. In other words, the time to repair increases each time with the number of repairs. Later on we will call this treatment method (α, β) *rule*.

2.7. Treatment method 7 — multiple (p, q) rule

Shaked and Shanthikumar (1986) introduce the multivariate imperfect repair concept. They consider a system whose components have dependent lifetimes and are subject to imperfect repairs respectively until they are replaced. For each component the repair is imperfect according to the (p, q) rule, i.e., at failure the repair is perfect with probability p and minimal with probability q. Assuming that n components of the system start to function at the same time 0, and no more than one component can fail at a time, they establish the joint distribution of the times to next failure of the functioning components after a minimal repair or perfect repair. They also derive the joint density of the resulting lifetimes of the components and other probabilistic quantities of interest, from which the distribution of the lifetime of the system can be obtained. Sheu and Griffith (1992) further extended this work. Later we will call this treatment method *multiple* (p, q) *rule*.

2.8. Others

Nakagawa and Yasui (1987) modeled imperfect PM in a way that in the steadystate, PM reduces the failure rate of a unit to a fraction of its value just before PM and during operation the failure rate climbs back up. He thinks that the portion by which the failure rate is reduced is a function of some resource consumed in PM and a parameter. That is, after PM the failure rate of the unit becomes

$$\lambda(t) = g(c_1, \theta) \cdot \lambda(x + T)$$

where the fraction reduction of failure rate $g(c_1, \theta)$ lies between 0 and 1, T is the time interval length between PM's, c_1 is amount of resource consumed in PM and θ is a parameter. This treatment method is different from the improvement factor method. The difference is that by improvement factor method maintenance makes system younger in terms of its age, that is, its age becomes younger.

Nakagawa (1986, 1988) uses two other methods to deal with imperfect PM for two sequential PM policies: (1) the failure rate after k PM becomes $a_k h(t)$ assuming that it was $h(t)$ in previous period where $a_k \geq 1$. That is, the failure rate increases with the number of PM's; (2) the age after k PM reduces to $b_k t$ when it was t before PM where $0 \leq b_k < 1$. That is, PM reduces the age. These two modeling methods will be called *reduction method* later. Besides, in investigating periodic PM models, Nakagawa and Yasui (1987) treat imperfect PM in a way that the age of the unit becomes x units of time younger by each PM and further suggests that the x is in proportion to the PM cost where x is less than or equal to the PM intervention interval. We will call it *x rule* later. Yak et al. (1984) think that maintenance may result in system failure (the worst maintenance) in modeling the MTTF and the availability of the system.

According to treatment methods, work on imper-

Table 1
Reference classification by treatment methods

Modeling method	References
(p, q) rule	Chan and Downs, 1978; Helvic, 1980; Nakagawa, 1979a,1980; Nakagawa and Yasui, 1987; Brown and Proschan, 1983; Fontenot and Proschan, 1984; Lie and Chun, 1986; Yun and Bai, 1987; Bhattacharjee, 1987; Rangan and Grace, 1989; Goel et al., 1991a,b; Sheu and Liou, 1992; Srivastava and Wu, 1993; Wang and Pham, 1996a,b,c
$(p(t), q(t))$ rule	Beichelt, 1980, 1981; Block et al., 1985, 1988; Abdel-Hameed, 1987a,b; Whitaker and Samaniego, 1989; Sheu, 1991, 1992; Sheu and Griffith, 1992; Sheu and Kuo, 1994; Sheu et al., 1993, 1995; Makis and Jardine, 1991; Iyer, 1992; Hollander et al., 1992; Sheu and Kuo, 1994
Improvement factor	Malik, 1979; Canfield, 1986; Lie and Chun, 1986; Jayabalan and Chaudhuri, 1992a,b,c,1995; Chan and Shaw, 1993; Suresh and Chaudhuri, 1994
Virtual age	Uematsu and Nishida, 1987; Kijima et al., 1988; Kijima, 1989; Makis and Jardine, 1993; Liu et al., 1995
Shock	Bhattacharjee, 1987; Kijima and Nakagawa, 1991; Kijima and Nakagawa, 1992; Sheu and Liou, 1992
(α, β) rule	Wang and Pham, 1996a,b,c
Multiple (p, q) rule	Shaked and Shanthikumar, 1986; Sheu and Griffith, 1992
Other	Nakagawa, 1979b,1980; Nakagawa, 1986, 1988; Subramanian and Natarajan, 1990; Nguyen and Murthy, 1981a,b,c; Yak et al., 1984; Yun and Bai, 1988; Dias, 1990; Subramanian and Natarajan, 1990; Zheng and Fard, 1991; Jack, 1991; Chun, 1992; Dagpunar and Jack, 1994; Zhao, 1994

Table 2
Age-dependent PM policy

Reference	PM	CM	Treatment method	Optimality criterion	Modeling tool	Planning horizon
Chan and Downs, 1978	imperfect	perfect	(p, q) rule	availability cost rate	semi-Markov	infinite
Nakagawa, 1979a	imperfect	perfect	(p, q) rule	cost rate	renewal theory	infinite
Beichelt, 1980	perfect	imperfect	$(p(t), q(t))$ rule	cost rate	renewal theory	infinite
Fontenot and Proschan, 1984	perfect	imperfect	(p, q) rule	cost rate	renewal theory	infinite
Block et al., 1988	perfect	imperfect	$p(t), q(t)$ rule	cost rate total cost	renewal theory finite	infinite
Rangan and Grace, 1989	perfect	imperfect	(p, q) rule	total cost	renewal theory	finite
Sheu, 1991, Sheu et al., 1993, 1995	perfect	imperfect	$(p(t), q(t))$ rule	cost rate (random cost)	renewal theory	infinite
Wang and Pham, 1996b	imperfect	imperfect	(p, q) rule (α, β) rule	cost rate availability	renewal theory	infinite

fect maintenance can be classified as in Table 1. From this table we can see that the (p, q) rule and $(p(t), q(t))$ rule are popular in treating imperfect maintenance. This is partly because these two rules make imperfect maintenance modeling mathematically tractable.

3. Imperfect maintenance models for various policies

3.1. Age-dependent PM policy

In the age-dependent PM model, a unit is preventively maintained at predetermined age T, or repaired at failure, whichever comes first. For this policy there are various imperfect maintenance models according to the conditions that either or both of PM and CM is imperfect. These models under the age-dependent PM policy and the extensions of this policy are summarized in Table 2.

One of the pioneer imperfect maintenance models for the age-dependent PM policy is due to Nakagawa (1979a). He considers three age-dependent PM models with imperfect PM and perfect or minimal repair at failure. He derives the expected maintenance cost rate and discusses the optimal maintenance policies in terms of PM interval time T.

Sheu et al. (1993) generalized the age-dependent PM policy where if a system fails at age $y < t$, it is subject to perfect repair with probability $p(y)$, or undergoes minimal repair with probability $q(y) = 1 - p(y)$. Otherwise, a system is replaced when the first failure after t occurs or the total operating time reaches age T $(0 \le t \le T)$, whichever occurs first. They discussed the optimal policy (t^*, T^*) to minimize the expected cost rate. This is a realistic PM model. Sheu et al. (1995) further extend this model. They assume that a system has two types of failures when it fails at age z and is replaced at the nth type 1 failure or first type 2 failure or at age T, whichever occurs first. Type 1 failure occurs with probability $p(z)$ and is corrected by minimal repair. Type 2 failure occurs with probability $q(z) = 1 - p(z)$ and is corrected by perfect repair (replacement). Using $(p(t), q(t))$ rule and random minimal repair costs, they derive the expected cost rate and also provide a numerical example.

3.2. Periodic PM policy

In the periodic PM policy, a unit is preventively maintained at fixed time intervals and repaired at intervening failures. The research under the periodic PM policy and extensions of this policy is summarized in Table 3. Liu et al. (1995) investigate an

Table 3
Periodic PM policy

Reference	PM	CM	Treatment method	Optimality criterion	Modeling tool	Planning horizon
Nakagawa, 1979a	imperfect	minimal	(p, q) rule	cost rate	renewal theory	infinite
Nakagawa, 1980	imperfect	perfect minimal	(p, q) rule x rule	cost rate	renewal theory	infinite
Beichelt, 1981	perfect	imperfect	$(p(t), q(t))$ rule	cost rate	renewal theory	infinite
Fontenot and Proschan, 1984	perfect	imperfect	(p, q) rule	cost rate	renewal theory	infinite
Nakagawa, 1986	imperfect	minimal	different failure rates	cost rate	renewal theory	infinite
Abdel-Hameed, 1987a	perfect	imperfect	$(p(t), q(t))$ rule	cost rate	stochastic process	infinite
Nakagawa and Yasui, 1987	imperfect	perfect	(p, q) rule	availability	renewal theory	infinite
Kijima et al., 1988	perfect	imperfect	virtual age	cost rate	renewal theory	infinite
Kijima and Nakagawa, 1991	imperfect	perfect	shock model	cost rate	renewal theory	infinite
Jack, 1991	perfect	imperfect	others	total cost	renewal theory	finite
Chun, 1992	imperfect	minimal	x rule	total cost	probability	finite
Sheu, 1992	perfect	imperfect	$(p(t), q(t))$ rule	cost rate	renewal theory	infinite
Liu et al., 1995	imperfect	minimal	virtual age	cost rate	renewal theory	infinite
Wang and Pham, 1996a	imperfect	imperfect	(p, q) rule (α, β) rule	cost rate availability	renewal theory	infinite

Table 4
Failure limit policy

Reference	PM	CM	Measure improved	Optimality criterion	Modeling tool	Planning horizon
Malik, 1979	imperfect	none	reliability	reliability	probability	infinite
Canfield, 1986	imperfect	none	failure rate	cost rate	renewal theory	infinite
Lie and Chun, 1986	imperfect	imperfect (p, q) rule	failure rate	cost rate	renewal theory	infinite
Jayabalan and Chaudhuri, 1992a	imperfect	minimal	failure rate	total cost	probability	finite
Jayabalan and Chaudhuri, 1992c	imperfect	minimal	age others	cost rate	probability	infinite
Jayabalan and Chaudhuri, 1992d	imperfect	none	age	total cost	probability	finite
Chan and Shaw, 1993	imperfect	perfect	failure rate	availability	probability	infinite
Suresh and Chaudhuri, 1994	imperfect		reliability and failure rate	total cost	probability	finite
Jayabalan and Chaudhuri, 1995	imperfect	minimal	age	total cost	renewal theory	finite

extended periodical PM model different from the classical one using the notation of the virtual age. They assume that a system receives (imperfect) PM every T time unit, intervening failures are subject to minimal repairs and the system is replaced every fixed number of PM's. Nakagawa (1986) studies a similar model but he assumes that PM is imperfect in a sense that after PM the failure rate will be changed.

3.3. Failure limit policy

This policy assumes that PM is performed only when the failure rate or reliability of a system reaches a predetermined level. Malik (1979) considers a reliable system and derives the PM schedule points so that the system works at or above the minimum acceptable level of reliability. Lie and Chun (1986) formulate a cost model where PM is performed whenever the system reaches the predetermined maximum failure rate. Jayabalan and Chaudhuri (1992a) have obtained the optimal replacement policy for a specific period of time where down time for installation and PM are assumed to be negligible. Jayabalan and Chaudhuri (1992b) consider down time for replacement as constant. As the systems ages, the successive downtime for PM interventions are expected to consume more time. To incorporate this, Jayabalan and Chaudhuri (1992b) assume that PM time is allowed to follow exponential distribution with increased downtime. Jayabalan and Chaudhuri (1995) present an algorithm to obtain optimal maintenance polices which require less computational time. The research under the failure limit category is summarized in Table 4.

3.4. Sequential PM policy

Nakagawa (1986, 1988) considers a sequential PM policy where PM is done at fixed intervals x_k where $x_k \le x_{k-1}$ for $k = 2, 3, \ldots$ This policy is very practical because most systems need to perform maintenance more frequent when the age increased. This PM policy is different from the failure limit policy in that it controls x_k lengths directly but the failure limit policy controls failure rate, age, reliability, etc., directly. The research under the sequential PM policy and its extensions is summarized in Table 5.

Table 5
Sequential PM policy

Reference	PM	CM	Treatment	Optimality criterion	Modeling tool	Planning horizon
Nakagawa, 1986	imperfect	minimal	different failure rates	cost rate	renewal theory	infinite
Nakagawa, 1988	imperfect	minimal	reduction (age and failure rate)	cost rate	renewal theory	infinite
Kijima and Nakagawa, 1992	imperfect	minimal	shock model	cost rate	renewal theory	infinite

3.5. Repair limit policy

When a system fails, the repair cost is estimated and repair is undertaken if the estimated cost is less than a predetermined limit; otherwise, the system is replaced. This is called repair cost limit policy in the literature. Yun and Bai (1987) and Wang and Pham (1996a) study the optimal repair cost limit policies under imperfect maintenance assumption.

A repair time limit replacement policy is proposed by Nakagawa and Osaki (see Nguyen and Murthy, 1981) in which a system is repaired at failure: if the repair is not completed within a specified time T, it is replaced by a new one; otherwise the repaired system is put into operation again where T is called the repair limit time. Nguyen and Murthy (1981) consider the repair time limit replacement policies with imperfect repair in which there are two types of repair — local and central repair. The local repair is imperfect while the central repair is perfect. The optimal policies are derived to minimize the expected cost rate for an infinite time span. They presented analytical results along with numerical examples. The research under this repair limit policy is summarized in Table 6.

3.6. Multicomponent systems

Imperfect maintenance models for multicomponent systems are summarized in Table 7. For a series system Zhao (1994) presents a series system availability model in which either minimal repair or perfect repair of all components can be modeled based on the work of Barlow and Proschan (1975). He assumes that the repaired component might not be as good as new and its lifetime may follow any

Table 6
Repair limit policy

Reference	CM before cost limit	CM after cost limit	Treatment	Optimality criterion	Modeling tool	Planning horizon
Beichelt, 1978, 1981	minimal	perfect	$(p(t), q(t))$ rule	cost rate	renewal theory	infinite
Nguyen and Murthy, 1981a	imperfect	perfect	others	cost rate	renewal theory	infinite
Yun and Bai, 1987	imperfect	perfect	(p, q) rule	cost rate	renewal theory	infinite
Yun and Bai, 1988	minimal	perfect	others	cost rate	renewal theory	infinite
Wang and Pham, 1996a	imperfect	imperfect	(p, q) rule (α, β) rule	cost rate availability	renewal theory	infinite

Table 7
Multicomponent systems subject to imperfect maintenance

Reference	PM	CM	Treatment	Optimality Criterion	Modeling tool	Horizon/configuration/policy
Shaked and Shanthikumar, 1986	none	imperfect	multiple (p, q) rule	none	renewal	infinite/arbitrary/
Subramanian and Natarajan, 1990	none	imperfect	other	reliability availability	stochastic process	infinite/two-unit standby/
Goel et al., 1991a,b	none	imperfect	(p, q) rule	availability	Markov process	infinite/two-unit standby/
Zheng and Fard, 1991	perfect	imperfect	other	cost rate	probability	infinite/arbitrary/ failure limit
Sheu and Griffith, 1991	none	imperfect	multiple $(p(t), q(t))$ rule	None	renewal theory	infinite/arbitrary/ age-dependent
Sheu and Liou, 1992	perfect	imperfect	$(p_1(t), \ldots, p_n(t))$ rule	cost rate	nonhomogeneous Poisson process	infinite/k-out-of-n/ age-dependent
Zhao, 1994	none	imperfect	other	availability	probability	infinite series
Sheu and Kuo, 1994	perfect	imperfect	$(p(t), q(t))$ rule	cost rate (random cost)	renewal theory	infinite/k-out-of-n/ age-dependent
Wang and Pham, 1996d	none	imperfect	(α, β)	availability	renewal theory	infinite/series/
Wang and Pham, 1996e	perfect	imperfect	(p, q) rule	availability cost rate	renewal theory	infinite/arbitrary/ age-dependent

distribution which can be different from that of old one after repair and obtain the mean limiting availability and mean system down and up time. In this model of series system, repair time is not negligible and thus it is practical. The related research is summarized in Table 7.

3.7. Others

Nguyen and Murthy (1981) discuss an imperfect PM policy where for a system whose most recent maintenance was CM, perform PM at age T_1; for a system whose most recent maintenance was PM, perform PM at age T_2. They treat imperfect PM in a way that after PM, the system has a different (worse) failure time distribution than after corrective maintenance (CM). Besides the age-dependent PM model in Section 3.1, Fontenot and Proschan (1984) discussed three other imperfect maintenance models using the (p, q) rule.

Jack (1991) considers a maintenance policy involving imperfect repairs on failure with replacement after N failures. Dagpunar and Jack (1994), Jack and Dagpunar (1994) consider to determine the optimal number of imperfect PM during a finite horizon assuming that the minimal repairs are made at any failures between PM's and the ith PM makes the age of a system x_i units of time younger (*x rule*), which is the same as the treatment method by Nakagawa (1980). Chun (1992) considers determination of the optimal number of periodic PM during a finite planning horizon using the same treatment method as in Nakagawa (1980), i.e., *x rule*.

Makis and Jardine (1992) considered a replacement policy in which a system can be replaced at any time at a cost c_0, and at the nth failure the system is either replaced at the cost c_0 or undergoes a imperfect repair at a cost $c(n, t)$ where t is the age of the system assuming that there has been no PM's. They use the $(p(n, t), q(n, t), s(n, t))$ rule to model imperfect repair. Makis and Jardine (1991, 1993) discuss the optimal replacement policy with imperfect repair at failure under a T-policy: a system is replaced each time at the first failure after some fixed time using the $(p(t), q(t), s(t))$ rule and the virtual age method respectively.

Block et al. (1993) introduced a generalized age replacement policy — repair replacement policy where systems are preventively maintained when a certain time has elapsed since their last repair. If the last repair was a perfect repair, this policy is essentially the same as age replacement policy. Srivastava and Wu (1993) consider an imperfect inspection model in which failures can only be detected with probability p, and also discuss the estimation of parameter p in this imperfect inspection model.

4. Concluding remarks

We have discussed various treatment methods and optimal maintenance policies of single- and multicomponent systems the rapidly growing research on the subject of imperfect maintenance in this paper. Today many researchers are pursuing the development of mathematical maintenance models to estimate the reliability measures and determine the optimum maintenance policies for multicomponent systems with imperfect maintenance. We, in deed, feel that these models would be valuable to maintenance engineers, designers, and practitioners if they are capable of incorporating information about the repair and maintenance strategy, the maintenance and inspection processes, the engineering management policies, the methods of failure detection, failure mechanisms, the environmental factors that justify the reasonableness of assumptions, and the applicability of a model in a given system environment and can give greater confidence in estimates based on small numbers of production data.

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