Graphical Representation of Two Mixed-Weibull Distributions

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Key Words — **Mixed-Weibull distribution, Probability plot, Graphical method**

Reader Aids —
Purpose: Tutorial and widen the state of the art
Special math needed for explanations: Elementary statistics
Special math needed to use results: Same
Results useful to: Reliability analysts

Summary & Conclusions — **A variety of shapes of 2-Weibull mixtures on Weibull probability paper (WPP) are explored and classified into 6 types of Cdf curves (A - F). Types B - D represent Cdf's composed of 2 well-mixed subpopulations; type F represents the Cdf composed of 2 very well-separated subpopulations; and types A & E are in between types B - D and type F. The Kao-Cran graphical parameter estimation method can not be applied to type A - C, E, F curves; we recommend that it not be applied to the type D curve. The theoretical basis is developed for the Jensen-Petersen graphical method for mixtures with well separated subpopulations, viz, type F curve. For type A & E curves, the method can be applied. But for type B - D curves, there is no theoretical basis for the method.**

1. INTRODUCTION

The pdf of lifetimes of units with more than one failure mode can be bimodal or multimodal [1 - 4]. Consequently, a mixed-Weibull distribution can be a good model for the lifetime of such units [5 - 7]. To apply the mixed-Weibull distribution the following questions must be answered:

- When can a mixed-Weibull distribution be applied?
- How many simple subpopulations are in the mixture?
- How can the parameters of the proposed mixed-Weibull distribution be determined? □

The graphical method is very popular in reliability data analysis due to its simplicity and visibility. This paper studies the graphical Cdf curves and estimates the parameter of mixed-Weibull distributions.

Section 2 describes the mixture of two Weibull distributions and its components. Section 3 studies the behavior of 2-Weibull mixture on WPP. A variety of shapes of 2-Weibull mixtures are explored and classified into 6 types. Section 4 presents 3 important features of a mixture with two well separated subpopulations. Section 5 discusses two existing methods for estimating the parameters graphically; one is recommended.

Notation

$Q(t)$, $Q_Z(z)$ Cdf of mixed Weibull distribution
$f_i(t)$, $Q_i(t)$ pdf, Cdf of subpopulation i, $i=1, 2$
$L_1(t)$, $L_2(t)$ tangent line drawn at the left, right end of the fitted Cdf curve
$\bar{\ }$ overbar: implies the complement, eg, $\bar{p}=1-p$
$\hat{\ }$ hat: implies a parameter estimate, eg $\hat{p}$ is an estimate of p
p, $\bar{p}$ mixing weight for subpopulation 1, 2; $0 \le p \le 1$
η_i, β_i scale, shape parameter of Q_i, $i=1, 2$; all are positive □

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

2. THE MIXED-WEIBULL DISTRIBUTION AND ITS COMPONENTS

The Cdf (unreliability) of a 2-Weibull mixture is:

$$Q(t) = p\cdot Q_1(t) + \bar{p}\cdot Q_2(t)$$

$$= p\cdot \mathrm{weif}(t/\eta_1;\ \beta_1) + \bar{p}\cdot \mathrm{weif}(t/\eta_2;\ \beta_2),\ t\ge 0. \quad (1)$$

A 2-Weibull mixture has 2 components. The shape of the mixture's pdf is influenced by β_1, β_2, p. If $\beta_1 \approx \beta_2$ and $\eta_1 \approx \eta_2$ then the mixture is composed of two well-mixed subpopulations, thus it is difficult to tell to which subpopulation an observation from the mixture belongs. If β_1 & β_2 are different, and the modes of the distributions are quite different, then the mixture is composed of two well separated subpopulations.

In practice, the degree of separation between the component pdf's is very important in:

- identifying the existence of a mixture,
- determining the number of subpopulations in the mixture,
- determining the parameters of each subpopulation and the mixing weights of the mixture.

For example, the simulation results of the estimator of p presented by Woodward et al [8,9] have lower s-bias and rms error when the separation of the component pdf's is large. To quantify the separation of two component pdf's, Woodward, et al [8] proposed a separation-measure, viz, the overlap area of $p\cdot f_1(t)$ & $\bar{p}\cdot f_2(t)$. However, we recommend using the overlap area of $f_1(t)$ & $f_2(t)$ [10].

3. WPP BEHAVIOR OF MIXED-WEIBULL DISTRIBUTION

The Cdf of a 1-Weibull distribution on WPP is a straight line. For the mixed-Weibull distribution, however, the Cdf is

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not a straight line on WPP, but some vestiges of linearity can be observed.

Notation & Assumptions

z	$(t/\eta_1)^{\beta_1}$
η	$(\eta_2/\eta_1)^{\beta_1}$
β	β_2/β_1; $\beta > 1$
Y	$\ln(-\ln[\overline{Q(t)}]) = \ln(-\ln[\overline{Q_Z(z)}])$
X	$\ln(z) = \beta_1 \cdot \ln(t) - \beta_1 \cdot \ln(\eta_1)$ □

Eq (1) becomes,

$$Q(t) = Q_Z(z) = p \cdot \text{expf}(z) + \bar{p} \cdot \text{weif}(z/\eta;\ \beta). \tag{2}$$

The X & Y are in a linear-linear coordinate system. The curve of $Q_Z(z)$ on WPP, compared with the curve of $Q(t)$, is shifted by $\beta_1 \cdot \ln(\eta_1)$ and is stretched (or squeezed) by a factor of β_1. Therefore the shapes of $Q(t)$ and $Q_Z(z)$ are the same if their mixing weights are the same. In other words, 2-Weibull mixtures have only one shape on WPP, if they have common values of β, η, p. Consequently, the 5 parameters in (1) can be reduced to 3 in studying the Cdf curves of 2-Weibull mixtures on WPP.

In order to explore the relationships among the Cdf curves of the mixture, the mixing weight, and the individual subpopulations, the following have been included in the Cdf plots, as shown in figure 1a:

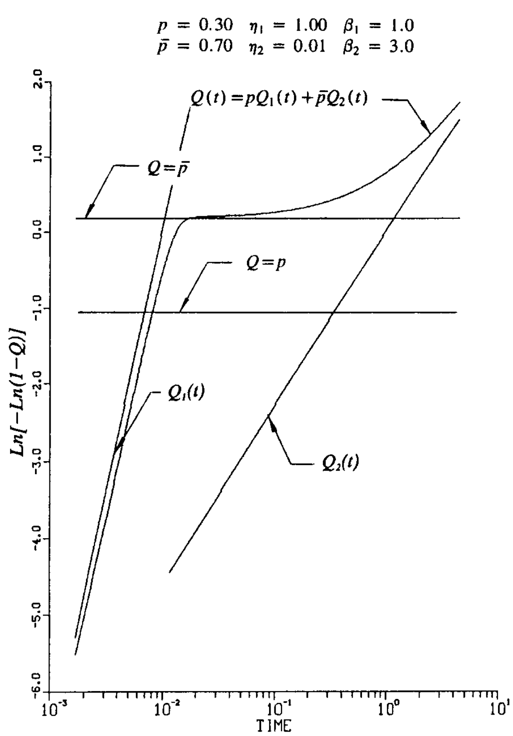

Figure 1a. Weibull Plot of a Type A Curve of a 2-Weibull Mixture

- the Cdf curves of the mixture,
- the Cdf's of the individual subpopulations,
- horizontal lines drawn at the levels of the underlying mixing weight, p & $\bar{p}$.

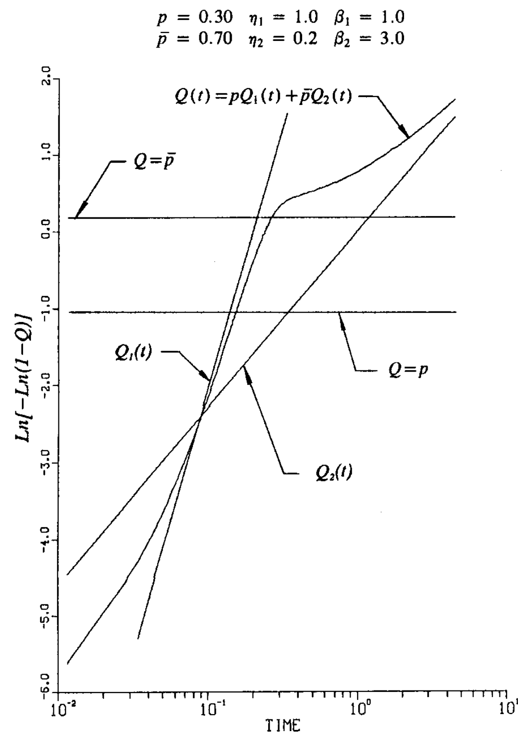

Figure 1b. Weibull Plot of a Type B Curve of a 2-Weibull Mixture

2-Weibull mixtures with the parameter combinations of $\beta = 1.0(0.5)8.0$, and $\eta = 0.001, 0.0015, 0.003, 0.006$, and similarly scaled numbers up to 1000 were plotted for $p = 0.3$. Figures 1a - 1f show the 6 typical shapes of Cdf curves on WPP, viz, types A - F. The curve type of each plot is plotted in figure 2, where the $\eta - \beta$ plane was partitioned into 7 regions (A - F, S). Region S represents the 2-Weibull mixtures whose Cdf curves are very close to a straight line. A 1-Weibull distribution can be classified as a special case of a 2-Weibull mixture with $\eta = 1$ and $\beta = 1$.

- Type A curve occurs when the component with larger β is located far to the left of the other pdf.
- Type B curve is like the type A curve, except that the overlap of the two pdf's of the type B curve is larger than that for type A.
- Type C curve occurs when the β of one component is much larger than the β of the other, and their η are very close.
- Type D curve occurs when the component with larger β is located to the right of the other pdf, and the η of the pdf with the larger β is slightly larger than that of the other pdf.
- Type E & F curves are close, with the larger β component's pdf located to the right of the other one, and the two component pdf's have a very small overlap. □

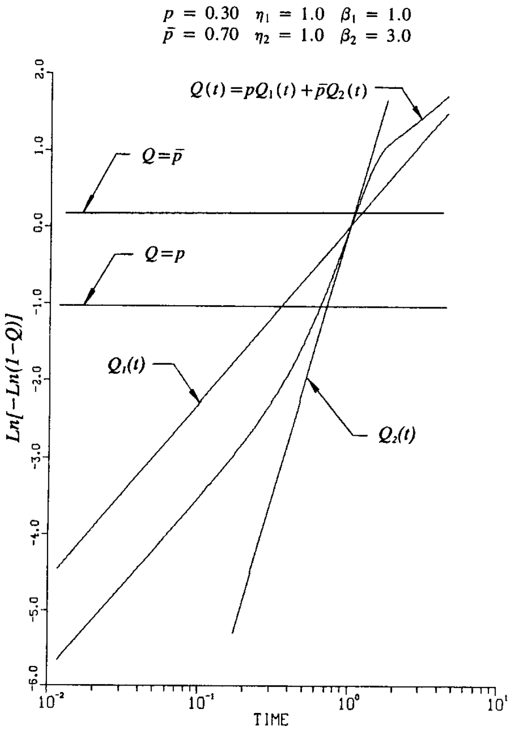

Figure 1c. Weibull Plot of a Type C Curve of a 2-Weibull Mixture

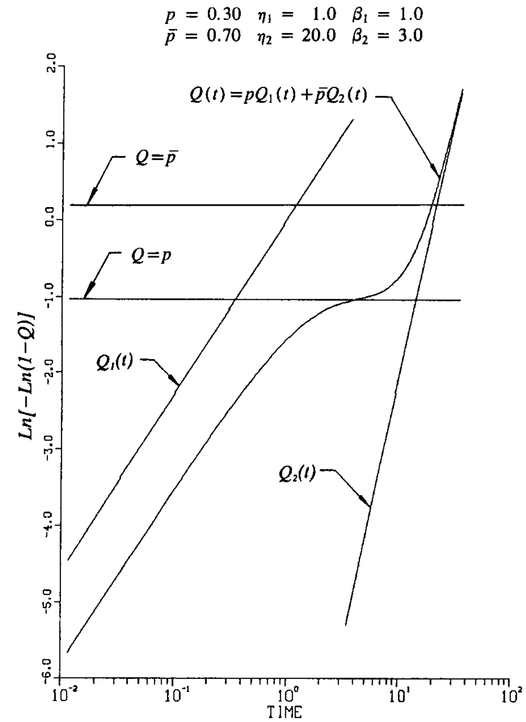

Figure 1e. Weibull Plot of a Type E Curve of a 2-Weibull Mixture

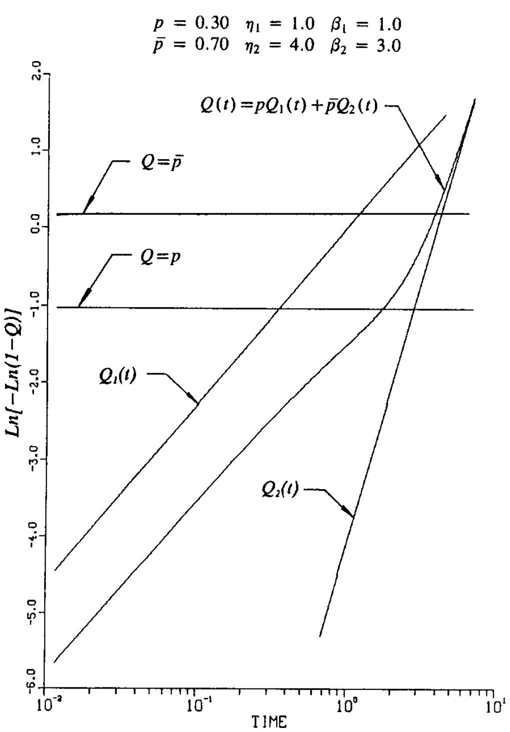

Figure 1d. Weibull Plot of a Type D Curve of a 2-Weibull Mixture

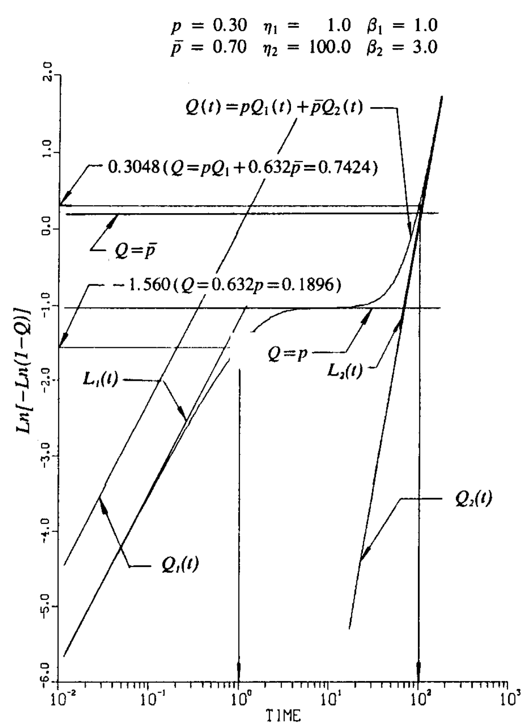

Figure 1f. Weibull Plot of a Type F Curve of a 2-Weibull Mixture

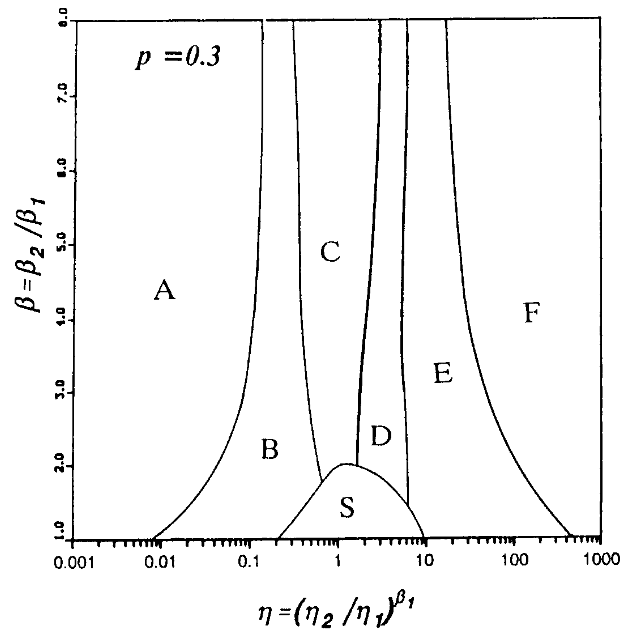

Figure 2. Map of Cdf Curve Types of 2-Weibull Mixtures [p = 0.3]

The parameter vector takes values in a continuous parameter space, ($0 \le p \le 1$, $\eta_1 > 0$, $\eta_2 > 0$, $\beta_1 > 0$, $\beta_2 > 0$). Therefore the Cdf curves which fall close to the boundary of the two regions possess the characteristics of both curve types.

One of the most important applications of probability plots is to identify an appropriate distribution type. In practical lifetime data analyses, a set of data is plotted on several probability papers, such as extreme value of the minima or of the maxima, Weibull, *s*-normal, or lognormal. It is possible that none of these plots fit the data well. Then the data analyst can look for other alternatives. If the plot of the data on WPP falls in one of the 6 typical shapes, then the 2-Weibull mixture can be a good model for this data set.

4. FEATURES OF A WELL-SEPARATED 2-WEIBULL MIXTURE

A well-separated, 2-Weibull mixture can be obtained as follows:

1. Fix β_1 & β_2 and let $\eta_1 \ll \eta_2$, or $\eta_1 \gg \eta_2$.
2. Fix $\eta_1 < \eta_2$ or $\eta_1 > \eta_2$, and increase β_1 & β_2.

Three important features of the Cdf curves on WPP (as proved in the appendix) when the two subpopulations are very well separated are:

1. The slope of the mixture Cdf tends to zero in a certain region,

$$dY/dX \to 0, \text{ for which } Q(e^x) \to p \text{ for some } x, \tag{3}$$

$$x = \ln(t) \tag{4}$$

$$Y = \ln(-\ln[\overline{Q(t)}]) = \ln(-\ln[\overline{Q(e^x)}]). \tag{5}$$

Eq (3)-(5) indicate that there exists an essentially horizontal portion on the Cdf curve, and its Y-axis reading is asymptotically equal to p.

2. The unreliability is:

$$Q(\eta_1) \to 0.632 \cdot p, \text{ for } t = \eta_1 < \eta_2, \tag{6a}$$

$$Q(\eta_2) \to p + 0.632 \cdot \bar{p}, \text{ for } t = \eta_2. \tag{6b}$$

Eq (6) implies that when p is known, then η_1 (or η_2) can be determined by entering the plot at the $0.632 \cdot p$ (or $p + 0.632 \cdot \bar{p}$) level horizontally, intersecting the Cdf curve, dropping down vertically, and reading η_1 (or η_2) from the X-axis directly.

3. The tangent lines drawn at the two ends of the Cdf curves are asymptotically parallel to the straight lines which represent the two individual subpopulations, respectively. For $\eta_1 < \eta_2$,

$$dY/dX \to \beta_1, \text{ as } x \to -\infty \ (t \to 0), \tag{7a}$$

$$dY/dX \to \beta_2, \text{ as } x \to +\infty \ (t \to \infty). \tag{7b}$$

These three features of the Cdf curves on WPP are the basis for the graphical method of parameter estimation of a 2-Weibull mixture when two subpopulations are well separated.

5. GRAPHICAL PARAMETER-ESTIMATION

The parameter estimation of mixed distributions is much more difficult than that of a single population. The difficulties are caused by the involvement of more unknown parameters in mixed distributions. Two graphical methods have been proposed to estimate the parameters in a 2-Weibull mixture:

- Kao-Cran: Kao [2] and Cran [11],
- Jensen-Petersen: Jensen & Petersen [1].

This section discusses their applicability.

5.1 *Kao-Cran Method*

The key step is to determine p.

1. Plot the sample data on WPP and fit a smooth curve by inspection. See figure 3.

2. At the left & right ends of the fitted Cdf curve, draw tangent lines, $L_1(t)$ & $L_2(t)$, respectively.

3. At the intersection of $L_2(t)$ with the *upper* borderline of the WPP, draw a vertical line whose intersection with $L_1(t)$ gives $\hat{p}$ which is read from the Y-axis.

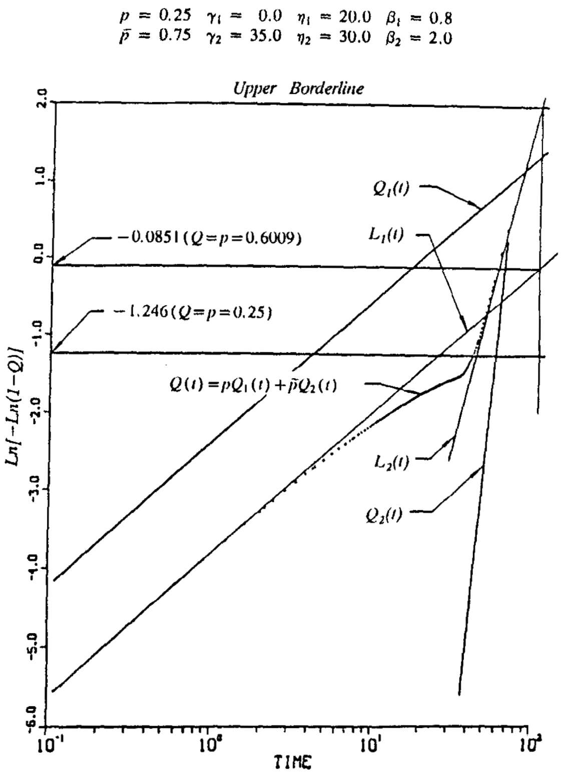

Figure 3. Kao-Chan Graphical Parameter Estimation

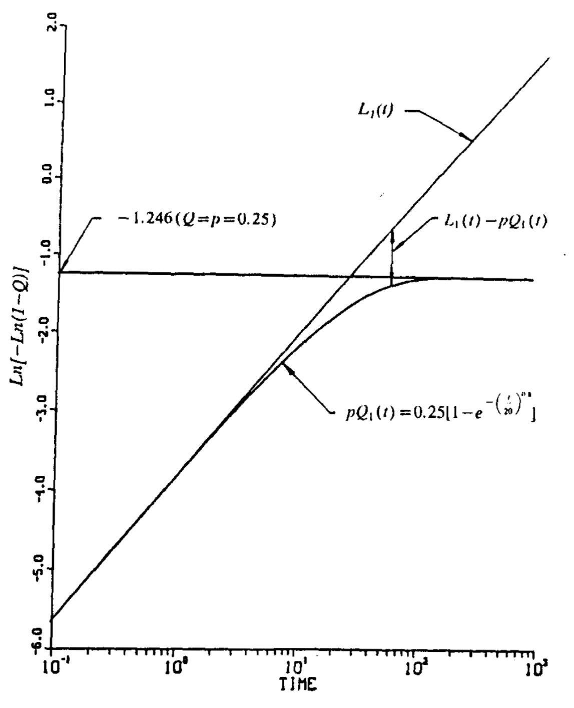

Figure 4. Error Caused By Kao-Cran Graphical Parameter Estimation

It is easy to see that the Kao-Cran method can not be applied to type A, B, C curves, and most type E & F curves, because the line tangent to the left end of the curve can not intersect the vertical line which is drawn from the intersection of the right-end tangent line and the upper borderline.

Now apply the Kao-Cran method to the type D curve. For example, in figure 3 the Cdf has been plotted using the same parameters in [11, example 2]. The estimate of the mixing weight is then $\ln[-\ln(\bar{Q})] = -0.0851$, or $\hat{p} = Q = 0.601$. This estimate is not acceptable when compared with the actual underlying mixing weight of $p = 0.25$. The big error is due to:

- As illustrated in figure 4, the tangent line $L_1(t)$ is a good approximation of $p \cdot Q_1(t)$ only for small values of t. Kao & Cran used $L_1(t)$ to approximate the curve $p \cdot Q_1(t)$ in the entire domain of $p \cdot Q_1(t)$. This leads to "an unbounded error" when $t \to \infty$.
- The estimate of p strongly depends on the upper borderline of WPP, and the upper borderline is chosen arbitrarily depending on the Y-axis scale range of the Weibull plotting paper used. □

Therefore, we do not recommend the Kao-Cran method.

5.2 *Jensen-Petersen Method*

The method is, briefly:

1. Plot the sample data on WPP and fit a smooth curve by inspection. See figure 1f.

2. Determine the place with the smallest slope on the Cdf curve, and read the corresponding p value from the Y-axis. p represents the mixing weight of the subpopulation located at the left. In this case $\ln[-\ln(\bar{Q})] = -1.0309$ where the slope of the Cdf curve in figure 1f is 0; consequently, $\hat{p} = Q = 0.30$.

3. Determine $\hat{\eta}_1$ & $\hat{\eta}_2$ by entering the Y-axis at $0.632 \cdot \hat{p}$ and $\hat{p} + 0.632 \cdot \bar{\hat{p}}$ horizontally; intersecting the Cdf curve and dropping down, then $\hat{\eta}_1$ & $\hat{\eta}_2$ can be read from the X-axis. For example, in figure 1f where $p = 0.30$, $0.632 \cdot p = 0.1896$, $\ln[-\ln(1-0.1896)] = -1.560$, $p + 0.632 \cdot \bar{p} = 0.7424$, and $\ln[-\ln(1-0.7424)] = 0.3048$. Then, $\hat{\eta}_1 = 1.0$ & $\hat{\eta}_2 = 100.0$ which are the values of the input parameters, η_1 & η_2 used in the Cdf plot.

4. Determine $\hat{\beta}_1$ & $\hat{\beta}_2$ from the slopes of the tangent lines which are drawn at each end of the Cdf curve. Then $\hat{\beta}_1 = 1$ and $\hat{\beta}_2 = 3$. □

From (3), (6), (7) and the discussion in section 4, it can be seen that the Jensen-Petersen method has a solid theoretical basis for mixtures with well separated subpopulations. For type F curve, it gives accurate estimates provided that the sample size of the data is large enough. For example, in figure 1f all 5 parameters can be determined with a negligible error. For type A & E curves in figures 1a & 1e, the Y-axis readings at the places with the smallest slope on the Cdf curves are very close to the underlying mixing weights of the subpopulations located at the left end of the mixture. Therefore, the Jensen-Petersen method can still be used to determine the parameters of each subpopulation, but not for type B - D curves. In these cases, maximum likelihood estimation [10] can solve the problem.

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APPENDIX

Proof of (3), (6), (7)

Let $R(t)$ be the reliability function of a 2-Weibull mixture, and $R_1(t)$ and $R_2(t)$ be the reliability functions of the subpopulations 1 and 2, respectively, then

$$R(t) = p\,R_1(t) + \bar{p}\,R_2(t) = \bar{Q}(t).$$

1. Using (4) and (5), and taking the derivative of Y with respect to t, yields

$$\frac{dY}{dt} = \frac{-1}{\ln[p\,R_1(t) + \bar{p}\,R_2(t)]} \cdot \frac{p\,f_1(t) + \bar{p}\,f_2(t)}{p\,R_1(t) + \bar{p}\,R_2(t)}. \tag{A-1}$$

Consider the effect of changing the values of η_1 and η_2:

First fix η_2, β_1, β_2 and t, let $\eta_1 \to 0$, then $p\,R_1(t) \to 0$ and $p\,f_1(t) \to 0$, therefore

$$\frac{dY}{dt} = \frac{-1}{\ln[\bar{p}\,R_2(t)]} \cdot \frac{\bar{p}\,f_2(t)}{\bar{p}\,R_2(t)}.$$

Second fix β_1, β_2 and t, let $\eta_2 \to \infty$, then $R_2(t) \to 1$ and $f_2(t) \to 0$, These lead to

$$\frac{dY}{dt} \to 0.$$

Let's rescale the abscissa, and let $x = \ln t$, then $t = e^x$. Taking the derivative of Y with respect to x yields

$$\frac{dY}{dx} = \frac{dY}{dt}\,\frac{dt}{dx} = \frac{dY}{dt}\,e^x. \tag{A-2}$$

Therefore,

$$\frac{dY}{dx} \to 0 \text{ when } \frac{dY}{dt} \to 0.$$

It may be seen, then, that $Q\,(e^x) \to p$ at the level where the slope of the 2-mixture Weibull plot is zero. It may also be seen that

$$R(t) = p\,R_1(t) + \bar{p}\,R_2(t) \to \bar{p}, \text{ as } \eta_1 \to 0 \text{ and } \eta_2 \to \infty,$$

because $R_1(t) \to 0$ and $R_2(t) \to 1$.

2. Calculate $Q(\eta_1)$ from

$$Q(\eta_1) = 1 - \left[p\,e^{-\left(\frac{\eta_1}{\eta_1}\right)^{\beta_1}} + \bar{p}\,e^{-\left(\frac{\eta_1}{\eta_2}\right)^{\beta_2}}\right].$$

Since when $\eta_1 \ll \eta_2$, $e^{-\left(\frac{\eta_1}{\eta_2}\right)^{\beta_2}} \to 1$, then

$$Q(\eta_1) \to 1 - \frac{p}{e} - \bar{p} = 0.632\,p.$$

Also

$$Q(\eta_2) = 1 - \left[p\,e^{-\left(\frac{\eta_2}{\eta_1}\right)^{\beta_1}} + \bar{p}\,e^{-\left(\frac{\eta_2}{\eta_2}\right)^{\beta_2}}\right],$$

and since when $\eta_1 \ll \eta_2$, $e^{-\left(\frac{\eta_2}{\eta_1}\right)^{\beta_1}} \to 0$, then

$$Q(\eta_2) \to 1 - \frac{\bar{p}}{e} = p + 0.632\,\bar{p}.$$

3. For fixed t it can be shown that $R_2(t) \to 1$ and $f_2(t) \to 0$ as $\eta_2 \to \infty$, and that $R_1(t) \to 0$ and $f_1(t) \to 0$ as $\eta_1 \to 0$. From (A-2)

$$\frac{dY}{dx} = \frac{dY}{dt}\,e^x = \frac{dY}{dt}\,t.$$

First fix the subpopulation 1 and move the subpopulation 2 far away to the right, ie, let $\eta_2 \to \infty$, then $R_2(t) \to 1$ and $f_2(t) \to 0$, which yields

$$\frac{dY}{dx} = \frac{-1}{\ln[\,p\,R_1(t) + \bar{p}]} \cdot \frac{p\,f_1(t)}{p\,R_1(t) + \bar{p}}\,t.$$

Using L'Hospital's rule yields

$$\frac{dY}{dx} \to \beta_1 \text{ as } t \to 0.$$

Similarly, fix the subpopulation 2 and move the subpopulation 1 to the left, ie, let $\eta_1 \to 0$, then $R_1(t) \to 0$ and $f_1(t) \to 0$, which yields

$$\frac{dY}{dx} = \frac{-1}{\ln[\bar{p}\,R_2(t)]} \cdot \frac{\bar{p}\,f_2(t)}{\bar{p}\,R_2(t)}\,t = \frac{-\beta_2 t^{\beta_2}}{\ln[\bar{p}\,R_2(t)]\eta_2^{\beta_2}}.$$

Using L'Hospital's rule yields

$$\frac{dY}{dx} \to \beta_2 \text{ as } t \to \infty.$$

The same conclusion can be obtained when $\eta_1 < \eta_2$, η_1 and η_2 are fixed, and β_1 or β_2 is increased.

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