Constrained Moment Formulations for Traffic Intensity Estimation in Single-Server Erlang Queues *

Eriky S. Gomes [1], Frederico R. B. Cruz [1],
Roberto C. Quinino [1], André L. F. Cançado [2]

[1]*Departamento de Estatística*
Universidade Federal de Minas Gerais
31270-901 - Belo Horizonte - MG, Brazil

[2]*Departamento de Estatística*
Universidade de Brasília
70910-900 - Brasília - DF, Brazil

eriky-tn@ufmg.br, fcruz@est.ufmg.br,
roberto@est.ufmg.br, acancado@unb.br

Abstract

This study investigates a single-server Erlang queueing system, in which the primary data observed consists of the number of customer arrivals occurring during individual service times. Such a setup arises naturally in applications where it is difficult to observe interarrival or service times directly, but counts of arrivals during service periods are available. We focus on the statistical inference of the traffic intensity, which is a key parameter characterizing the behavior of the queue. Specifically, we analyze the maximum likelihood estimator (MLE) for the traffic intensity based on the observed counts and show that this estimator follows a discrete distribution closely related to a right-truncated negative binomial distribution. Building on this result, we obtain closed-form expressions for the first two central moments of the MLE, namely, the mean and variance. Additionally, we demonstrate that these expressions converge asymptotically to the corresponding moments of a standard negative binomial distribution, reinforcing the consistency and interpretability of our estimator in large-sample settings.

Keywords: Queueing, $M/E_r/1$ queue, maximum likelihood estimation.

*Under review, 2025. DOI: doi. The final publication is available at somewhere.