Statistical Inference of Change Point in $M/M/1$ Queueing System with Balking *

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Abstract

This study investigates the estimation of change point and associated traffic intensities in an $M/M/1$ queueing system considering balking behavior, where customers may choose not to enter the queue depending on the system's current state. Real-world service systems often experience changes in traffic intensity due to changing arrival rates, and detecting such changes is crucial for efficient resource management. The article explores both classical and Bayesian approaches to estimate the change point and the associated traffic intensities before and after the change. Maximum Likelihood Estimators (MLEs) and Likelihood Ratio (LR) tests are developed for classical inference. In the Bayesian framework, gamma and inverted beta priors are used to derive posterior distributions and Bayes estimators. Extensive Monte Carlo simulations demonstrate the accuracy and robustness of the proposed estimators across varying sample sizes and parameter values. Furthermore, the Bayes factor is employed to compare models with and without change points. The results reveal that Bayesian estimators often outperform classical methods. Additionally, a practical illustration of the methodology is provided, demonstrating its effectiveness in parameter estimation and model comparison. This work contributes to adaptive queueing theory by offering analytical tools for dynamic systems impacted by customer decision-making under uncertainty.

Keywords: $M/M/1$ queueing system; balking behavior; change point estimation; maximum likelihood estimation (MLE); likelihood ratio test; Bayesian inference; Bayes factor.