Optimal Brown-Proschan Repair Policy for Repairable Systems Using the Discrete Weibull Failure Distribution and Markov Chains *

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Abstract

A repairable system operates under a maintenance strategy involving scheduled preventive maintenance and corrective repair actions following failures. This study develops an exact analytical methodology using Markov chains with discrete states and time to calculate the expected number of failures $\mathbb{E}[N(t)]$ under three repair policies: Minimal Repair, Perfect Repair, and the Brown-Proschan (BP) model. In the BP model, a failed unit is restored to "as good as new" condition with probability p (perfect repair) or to "as bad as old" condition with probability $1-p$ (minimal repair). Assuming failures follow a discrete Weibull distribution, we construct Markov chain models that provide exact analytical solutions for the expected number of failures until the next preventive maintenance (PM). The methodology enables exact calculation of reliability metrics essential for maintenance optimization and provides a foundation for cost analysis in discrete-time repairable systems. A case study presents the application of the Brown–Proschan model to the maintenance of trucks used for mineral exploration, extraction, and processing.

Keywords: Reliability; minimal repair; perfect repair; Brown-Proschan model; discrete Weibull distribution