# Parameter Estimation For Mixed-Weibull Distribution

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***SUMMARY & CONCLUSIONS***

In reliability engineering, it is known that electrical and mechanical equipment usually have more than one failure mode or cause. It has been recognized for more than three decades that the mixed Weibull distribution is an appropriate distribution to use in modeling the lifetimes of the units that have more than one failure cause. However, due to the lack of a systematic statistical procedure for fitting an appropriate distribution to such a mixed data set, it has not been widely used. A mixed Weibull distribution represents a population that consists of several Weibull subpopulations. In this paper, a new approach is developed to estimate the mixed-Weibull distribution's parameters. At first, the population sample data are split into subpopulation data sets over the whole test duration by using the posterior belonging probability of each observation to each subpopulation. Then, with the new concepts of Fracture Failure and Mean Order Number, the proposed approach combines the Least-Squares method with Bayes' Theorem, takes advantage of the parameter estimation for single Weibull distribution to each derived subgroup data set, and estimates the parameters of each subpopulation. The proposed approach can also be applied for complete, censored, and grouped data samples. Its superiority is particularly significant when the sample size is relatively small and for the case in which the subpopulations are well mixed. A numerical example is given to compare the proposed method with the conventional plotting method of subpopulation separation. It turns out that the proposed method yields more accurate parameter estimates.

*NOTATION*

| | |
|---|---|
| $f(t)$ | Probability density function, *pdf*, of a mixed population |
| $f_j(t)$ | Probability density function, *pdf*, of $j$th subpopulation, $j = 1, 2$ |
| $\beta_j$ , $\eta_j$ | Weibull shape and scale parameters of $f_j(t)$ |
| $p, q$ | Mixing weight for Subpopulation 1 and 2, $p \in (0, 1), p+q = 1$ |
| $P_j(t_i)$ | Posterior belonging probability |
| $MON_j(t_i)$ | Mean Order Number |
| $MR_j(t_i)$ | Median Rank |
| $Y_j(i)$ | $\log_e\{-\log_e[1- MR_j(t_i)]\}$ |
| $X(i)$ | $\log_e (t_i)$ |
| $\rho_1$ , $\rho_2$ | Correlation coefficients, $\rho = \rho_1 + \rho_2$ |

## 1. *INTRODUCTION*

The Weibull distribution has been used to model times-to-failure data successfully. However, when a product has two or more failure modes or causes; e.g., both early failures and chance failures might be involved in a burn-in test, the appropriate mixed-Weibull distribution must be used.

If the population consists of a mixture of two independent subpopulations with no correlation and each subpopulation has its own unique failure mode and distribution, then the lifetime distribution for the mixed population can be expressed by

$$f(t) = p f_1(t) + q f_2(t), \tag{1}$$

where

$f(t)$ = *pdf* of the mixed population,

$f_j(t)$ = *pdf* of the $j$th subpopulation, $j = 1, 2$,

$p$ = mixing weight, $p \in (0, 1)$,

and

$p + q \equiv 1$.

Usually, a subpopulation can be described by a single Weibull distribution; i.e.,

$$f_j(t) = \frac{\beta_j}{\eta_j}\left(\frac{t}{\eta_j}\right)^{\beta_j - 1} \exp\left[-\left(\frac{t}{\eta_j}\right)^{\beta_j}\right], \quad j = 1, 2. \tag{2}$$

Therefore, Eq. (1) becomes a mixed two-Weibull distribution with five parameters, $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$.

Jiang and Kececioglu (Ref. 1) found that there are six typical patterns of mixed, two-Weibull *CDF* curves on Weibull Probability Paper (WPP). In practice, if the plot of failure data on WPP falls in one of those six typical shapes, then a mixed two-Weibull distribution might be a good model to use for the analysis of the failure data. The question is how to fit the mixed Weibull distribution to the failure data or how to estimate the distribution's parameters properly.

Actually, the mixed Weibull distribution has been recognized as a candidate model of multi-mode failures in reliability engineering for more than three decades. It has not been widely used, because it is difficult to estimate the distribution parameters. This paper will present a new algorithm which combines the Least-Squares method with Bayes Theorem by introducing new concepts. The objective of the paper is to find an easy way to get accurate estimates of parameters or, at least to stimulate some new ideas to find a better estimation method later.

## 2. *PARAMETER ESTIMATION FOR A MIXED TWO-WEIBULL DISTRIBUTION*

### 2.1 *Current Methods*

The parameter estimation for a mixed distribution is much more difficult than that for a single distribution. The difficulties are partly caused by the involvement of more unknown parameters.

For a mixed, two-Weibull distribution, five parameters need to be estimated. Theoretically, the "best" way is to use the physics-of-failure analysis, classify each failure data point into a different subpopulation by its failure mode, and thus analyze each subpopulation separately. This additional failure analysis is usually costly, time-consuming, and impossible in most engineering practices. Sometimes, engineers even need to use statistical solutions instead of physics-of-failure analysis to identify the failure modes; e.g., engineers have to make a decision, based on a simple analysis of failure data.

Currently, two major estimation methods are used for the mixed, two-Weibull distribution: the graphical method (Refs. 1-3) and the MLE method (Ref. 4). The graphical parameter estimation method is very popular for the mixed Weibull due to its simplicity and visibility. Another reason for still using graphic estimates is that, so far, there is no other easy way to get reasonable estimates. This method is useful for the well-separated subpopulation cases. It depends on visual inspection of the data plots, which fails in most well-mixed cases. Also, it is hard to use for small sample sizes which is the case engineers often encounter. However, since it can be quickly carried out, the graphical method can provide initial estimates of the population parameters.

Maximum Likelihood Estimation (MLE) is preferred by statisticians because the MLE estimate has excellent statistical characteristics. It finds simultaneously all parameters that maximize the likelihood function of the observed sample. For a mixed Weibull population, the MLE is very complex. The Expectation and Maximization (EM) algorithm (Ref. 4) is recommended to solve the MLE for the mixed Weibull distribution. However, the calculations (iterations) may not always converge and multiple local maxima occur in all MLE algorithms. It has to be pointed out that, for small size samples, the MLE estimates tend to be highly biased and should be used very carefully.

The method presented here takes advantage of both physics-of-failure analysis results, which takes advantage of the single-Weibull analysis approach plus the MLE method, which uses every failure point of both subpopulations. The new method tries to split each failure point into two. By calculation, it theoretically separates the data sample into two subsamples corresponding to two subpopulations, respectively. Each subpopulation will fully use the information of the whole sample. Also the single-Weibull analysis approach can be applied to estimate the parameters of each subpopulation separately.

### 2.2 *The Application of Bayes' Theorem*

If a reliability life test is carried out on $N$ units of a product which has two failure modes, a times-to-failure sample $\{t_i,\ i = 1, 2, \ldots, N\}$ is obtained. Assume that

$$t_1 < t_2 < \cdots\cdots < t_N .$$

At time $t_i$, a failure is observed. To split this failure point rationally, the concept of belonging probability, $P_j(t_i)$, which is the posterior probability that this failure belongs to the $j$th subpopulation ($j = 1, 2$), is introduced by Kamath (Ref. 5), and Kececioglu and Sun (Ref. 6). By definition

$$P_j(t_i) = P\left\{T \in f_j(t) \middle| t_i - \tfrac{1}{2}\Delta t < T < t_i + \tfrac{1}{2}\Delta t\right\},\quad j = 1, 2,\ i = 1, 2, \ldots, N. \tag{3}$$

Applying Bayes' Theorem, yields

$$P_j = \frac{P\left\{t_i - \frac{1}{2}\Delta t < T < t_i + \frac{1}{2}\Delta t \middle| T \in f_j(t)\right\}\cdot P\left\{T \in f_j(t)\right\}}{\sum_j P\left\{t_i - \frac{1}{2}\Delta t < T < t_i + \frac{1}{2}\Delta t \middle| T \in f_j(t)\right\}\cdot P\left\{T \in f_j(t)\right\}}, \tag{4}$$

where

$$P\{\,T \in f_1(t)\,\} = p,$$

and

$$P\{\,T \in f_2(t)\,\} = 1 - p = q.$$

Then, the probability that a failure occurring at time $t_i$ belongs to Subpopulation 1 is

$$P_1(t_i) = \frac{p f_1(t_i)}{p f_1(t_i) + q f_2(t_i)} = \frac{p f_1(t_i)}{f(t_i)},\quad i=1, 2, \ldots, N. \tag{5}$$

Similarly, the probability that a failure occurring at time $t_i$ belongs to Subpopulation 2 is

$$P_2(t_i) = \frac{q f_2(t_i)}{p f_1(t_i) + q f_2(t_i)} = \frac{q f_2(t_i)}{f(t_i)},\quad i=1, 2, \ldots, N. \tag{6}$$

Note that for each failure point, the sum of all belonging probabilities must be unity; i.e.,

$$P_1(t_i) + P_2(t_i) = 1,\ \forall\, i .$$

### 2.3 *Determination of the Subsamples*

Without doing any physics-of-failure analysis, there is no way to tell exactly from which subpopulation an observation comes. By means of the posterior belonging probabilities, the failure occurring at $t_i$ can be divided into two portions: $100 \cdot P_1(t_i)$ percentage of this failure belongs to Subpopulation 1 and $100 \cdot P_2(t_i)$ percentage belongs to Subpopulation 2. It can be claimed that there were $P_1(t_i)$ failures expected at time $t_i$ if only Subpopulation 1 was put in test with the sample size of $N \cdot p$. In other words, $P_1(t_i)$ can be considered as the failed unit number at time $t_i$ in Subpopulation 1 and $P_2(t_i)$ the failed unit number of Subpopulation 2 at time $t_i$. Pooling these "fractional failures" versus their corresponding occurrence times under the same subpopulation yields the following two subsamples:

Subsample 1: $\{(t_1, P_1(t_1)), (t_2, P_1(t_2)), \ldots, (t_N, P_1(t_N))\}$;
Subsample 2: $\{(t_1, P_2(t_1)), (t_2, P_2(t_2)), \ldots, (t_N, P_2(t_N))\}$.

### 2.4 *Application of the Least-Squares Method*

If two different products, one of sample size $N \cdot p$ and another of $N \cdot q$ (total is $N$), respectively, are put into a reliability life test independently, two lifetime samples are obtained. In Test 1, $P_1(t_1)$ units fail at time $t_1$, $P_1(t_2)$ units fail at time $t_2$, ..., $P_1(t_N)$ units fail at time $t_N$. Similarly, $P_2(t_1)$ units fail at time $t_1$, $P_2(t_2)$ units fail at time $t_2$, ..., $P_2(t_N)$ units fail at time $t_N$ in Test 2. For each subpopulation, its corresponding subsample can be seen as a grouped data sample. The only difference is that the failure number at each failure point is not an integer. For each subsample, the conventional estimation method, the Rank Regression method, for single-Weibull distribution can be used to estimate the parameters of each subpopulation. The Mean Order Number (*MON*) of the *i*th failure in the *j*th subpopulation will be

$$MON_1(t_i) = \sum_{k=1}^{i} P_1(t_k), \quad i = 1, 2, \ldots, N, \tag{7}$$

and

$$MON_2(t_i) = \sum_{k=1}^{i} P_2(t_k), \quad i = 1, 2, \ldots, N. \tag{8}$$

The corresponding Median Ranks, $MR_j(t_i)$, can be calculated as follows:

Subpopulation 1

$$MR_1(t_i) = \frac{MON_1(t_i)}{MON_1(t_N) + 0.4}, \tag{9}$$

Subpopulation 2

$$MR_2(t_i) = \frac{MON_2(t_i)}{MON_2(t_N) + 0.4}. \tag{10}$$

Therefore, the following two paired sets will be obtained:

Subpopulation 1

$\{(t_1, MR_1(t_1)), (t_2, MR_1(t_2)), \ldots, (t_N, MR_1(t_N))\}$,

and Subpopulation 2

$\{(t_1, MR_2(t_1)), (t_2, MR_2(t_2)), \ldots, (t_N, MR_2(t_N))\}$.

Having these two data sets, the least-squares method can be used to determine the posterior parameters. The *CDF*, or unreliability, for a Weibull distribution can be written in the form of

$$\log_e\left\{\log_e \frac{1}{1 - MR_j(t_i)}\right\} = \beta_j \log_e t_i - \beta_j \log_e \eta_j, \tag{11}$$

or in the linearized form of

$$Y_j(i) = \beta_j X(i) + b_j, \tag{12}$$

where

$Y_j(i) = \log_e\{-\log_e[1 - MR_j(t_i)]\}$,
$X(i) = \log_e t_i$,
$b_j = -\beta_j \log_e \eta_j$.

Finally, applying the least-squares method, the distribution parameters are given by

$$\hat{\beta}_j = \frac{\sum_{i=1}^{N} X(i) Y_j(i) - \frac{1}{N}\left(\sum_{i=1}^{N} X(i) \cdot \sum_{i=1}^{N} Y_j(i)\right)}{\sum_{i=1}^{N} X^2(i) - \frac{1}{N}\left(\sum_{i=1}^{N} X(i)\right)^2}, j = 1, 2, \tag{13}$$

$$\hat{b}_j = \frac{1}{N}\sum_{i=1}^{N} Y_j(i) - \hat{\beta}_j \frac{1}{N}\sum_{i=1}^{N} X(i), \qquad j = 1, 2, \tag{14}$$

and

$$\hat{\eta}_j = \exp\left(-\frac{\hat{b}_j}{\hat{\beta}_j}\right), \qquad j = 1, 2. \tag{15}$$

On the other hand, the posterior mixing weight can be obtained from

$$\hat{p} = \frac{MON_1(t_N)}{N} = \frac{1}{N}\sum_{i=1}^{N} P_1(t_i), \tag{16}$$

or

$$\hat{p} = 1 - \hat{q} = 1 - \frac{MON_2(t_N)}{N} = 1 - \frac{1}{N}\sum_{i=1}^{N} P_2(t_i). \tag{17}$$

Note that the estimate given by Eq. (16) or (17) also satisfies the Maximum Likelihood Equation. For the mixed, two-Weibull distribution, the Maximum Likelihood Function is

$$L = \prod_{i=1}^{N} f(t_i) = \prod_{i=1}^{N}\left[p \cdot f_1(t_i) + q \cdot f_2(t_i)\right],$$

or

$$l = \log_e(L) = \sum_{i=1}^{N} \log_e\left[p \cdot f_1(t_i) + q \cdot f_2(t_i)\right].$$

Taking the partial derivative with respect to $p$, yields

$$\frac{\partial l}{\partial p} = \sum_{i=1}^{N} \frac{f_1(t_i) - f_2(t_i)}{f(t_i)}.$$

Substituting Eqs. (5) and (6) into it, yields

$$\frac{\partial l}{\partial p} = \frac{1}{p}\sum_{i=1}^{N} P_1(t_i) - \frac{1}{q}\sum_{i=1}^{N} P_2(t_i).$$

Then, substituting Eqs. (16) and (17) into this equation, yields

$$\frac{\partial l}{\partial p} = N \cdot \frac{p}{p} - N \cdot \frac{q}{q} = 0.$$

### *2.5 Algorithm*

The original estimation problem is that of having a data sample from a mixed population, which has a *pdf* given in Eqs. (1) and (2), and then of estimating the five parameters, $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$, such that the distribution fits the data "best".

From the mathematical point of view, the problem is to find the five unknowns from the given data set $\{t_i\ ;\ i = 1,\ 2, \ldots, N\}$ So, five relations (equations) among unknown parameters and given data need to be constructed to solve for the five unknowns.

From previous discussion, the belonging probabilities are completely determined if the distribution parameters and the data set are known. This means the belonging probabilities, $P_j(t_i)$, are a function of the parameters, $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$, and of the data set, $\{t_i\ ;\ i = 1,\ 2, \ldots, N\}$, only. From Eqs. (7) through (17), it is easy to see that $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$ are functions of the belonging probabilities, $P_j(t_i)$, and the data set, or

$$\beta_j = g_{1j}(\beta_1, \eta_1, \beta_2, \eta_2, p, t_i), \qquad j = 1, 2, \qquad (18)$$

$$\eta_j = g_{2j}(\beta_1, \eta_1, \beta_2, \eta_2, p, t_i), \qquad j = 1, 2, \qquad (19)$$

$$p = g_3(\beta_1, \eta_1, \beta_2, \eta_2, p, t_i). \qquad (20)$$

Note that Eqs. (18), (19) and (20) actually are five equations with respect to $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$ (unknowns), including the sample data $\{t_i\ ;\ i = 1,\ 2, \ldots, N\}$ (given). Theoretically, the least-squares parameter estimates would be obtained by solving these five equations directly. Obviously, it is impossible to get analytical solutions in practice. Normally, an alternative way is to use the iterative technique to solve the equations numerically. However, if the iterative procedure is applied directly, the calculation process may not always converge or may converge very slowly. Also the result is sensitive to the initial value and the quality of the data set. In practice, the quality of the data is not always good. Sometimes, engineers may face "dirty" data. In these cases, Eqs. (18), (19) and (20) may not have any solution or give very low quality estimates. However, the idea of the least-squares method is to find the "best" fit. Regardless of how good or bad the quality of the data, the "best" fitting line always exists. According to the least-squares principle, the "best' fitting line minimizes the residual variation around the line. Generally, the correlation coefficient, $\rho$, provides a good measure of how well the line fits the data. The larger the absolute value of $\rho$ is, the better the fitted line is. Based on this discussion, the "best" parameter estimates can be obtained by employing the least-squares principle to iterate on the $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$ values to minimize the deviations from the points to the line or maximize the correlation coefficient.

With any chosen set of $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$, the correlation coefficients of $\rho_1$ and $\rho_2$ are determined from Eqs. (5) through (11), and

$$\rho_j = \frac{\sum_{i=1}^{N} X(i) \cdot Y_j(i) - \frac{1}{N}\left[\sum_{i=1}^{N} X(i) \cdot \sum_{i=1}^{N} Y_j(i)\right]}{\left[\sum_{i=1}^{N} X^2(i) - \frac{1}{N}\left(\sum_{i=1}^{N} X(i)\right)^2\right] \cdot \left[\sum_{i=1}^{N} Y_j^2(i) - \frac{1}{N}\left(\sum_{i=1}^{N} Y_j(i)\right)^2\right]}, \quad j = 1, 2. \qquad (21)$$

Since two lines are simultaneously fitted to two subsamples from two Weibull subpopulations and every parameter has an effect on both correlation coefficients, the sum of the squares of these two correlation coefficients might be the "best" measure for the degree of fitting. Note that in the mixed two-Weibull case, the correlation coefficient is always positive, or $\rho_j > 0, j = 1, 2$. So, the sum of two coefficients, instead of the sum of the squares, can be simply used for the measure of the degree of fitting, or

$$\rho = \rho_1 + \rho_2. \qquad (22)$$

Therefore, employing the iterative procedure, the estimates of $\beta_1$, $\eta_1$, $\beta_2$, $\eta_2$, and $p$ can be obtained by maximizing the value of $\rho$, starting from a proper initial point ($\beta_1^0$, $\eta_1^0$, $\beta_2^0$, $\eta_2^0$, $p^0$). It is recommended to use the graphical estimates as the initial point to save search time. In the authors' experience the function of $\rho(\beta_1,\ \eta_1,\ \beta_2,\ \eta_2,\ p)$ has always displayed unimodal behavior. So, any nonlinear programming algorithm may be incorporated easily to handle this problem. The proposed computing flow chart is given in Fig. 1. Since every computing step is in closed form, this method is easy to program.

## EXAMPLE

Given are the life test data of Table 1. If the mixed two-Weibull distribution is used to represent the times-to-failure distribution, determine the mixed population's parameters using the proposed approach and the graphic method. Conduct the K-S goodness-of-fit test to compare the results.

**TABLE 1** – Failure data from a life test for Example.

| Failure order, *i* | Times to failure |
|---|---|
| 1 | 3.0 |
| 2 | 28.5 |
| 3 | 71.6 |
| 4 | 91.1 |
| 5 | 129.1 |
| 6 | 157.8 |
| 7 | 188.9 |
| 8 | 226.1 |
| 9 | 278.0 |
| 10 | 367.2 |

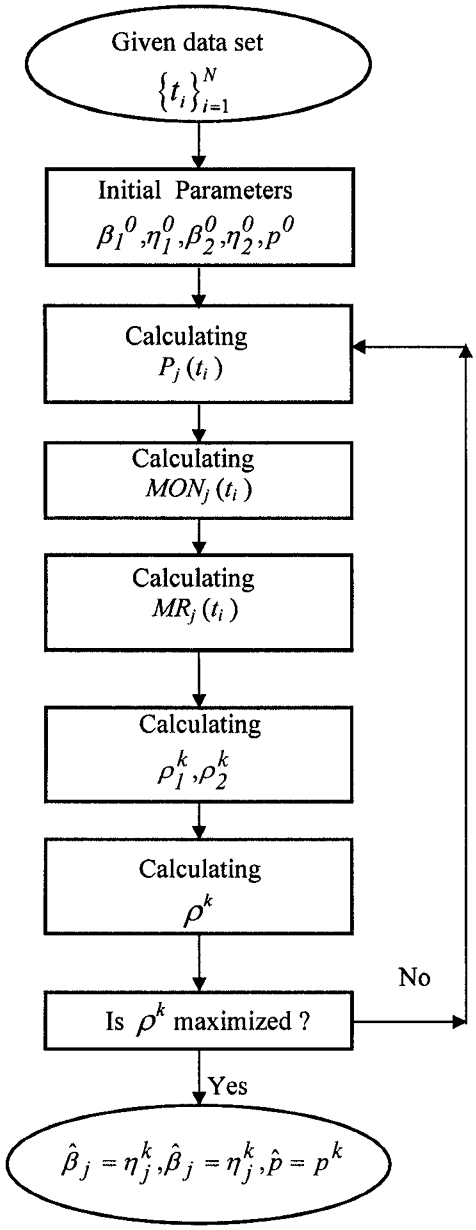


**Fig. 1** – The proposed computing flow chart.

## SOLUTIONS TO EXAMPLE

1. Graphic Estimation.

The separation plotting method (Ref. 2, pp. 531-579) is used to get the graphic estimation. The parameter estimates are found to be

$$\hat{p} = 0.3,\ \hat{\beta}_1 = 0.6,\ \hat{\eta}_1 = 39.4,\ \hat{\beta}_2 = 2.3,\ \hat{\eta}_2 = 233.5.$$

2. The Proposed Estimation.

To start the proposed approach, use the graphic estimates as the initial values of $\beta_1^0$, $\eta_1^0$, $\beta_2^0$, $\eta_2^0$, and $p^0$. Using the algorithm developed in this paper and shown yields (the calculation details are omitted here)

$$\hat{p} = 0.3,\ \hat{\beta}_1 = 0.5,\ \hat{\eta}_1 = 50.0,\ \hat{\beta}_2 = 1.9,\ \hat{\eta}_2 = 193.$$

3. Comparison

A comparison is made by conducting the Kolmogorov-Smirnov (K-S) goodness-of-fit test on the parameter values obtained by the graphical method and the proposed approach, as given in Table 2. It may be seen that the proposed approach yields a value of $D_{MAX}$ smaller than that of the graphical method.

**TABLE 2** – K-S goodness-of-fit test on the parameter estimates for the Example.

| Times to Failure, $t_i$ | $D^*$ (Graphic) | $D$ (Proposed) |
|---|---|---|
| 3.0 | 0.038181 | **0.032322** |
| 28.5 | 0.017825 | 0.015671 |
| 71.6 | 0.015086 | 0.019961 |
| 91.1 | 0.065898 | 0.012316 |
| 129.1 | 0.061995 | 0.020648 |
| 157.8 | 0.073926 | 0.019451 |
| 188.9 | **0.075794** | 0.017055 |
| 226.1 | 0.064070 | 0.014820 |
| 278.0 | 0.035371 | 0.012674 |
| 367.2 | 0.009758 | 0.003933 |

* $D = | Q_O(t_i) - Q_E(t_i) |$,

where

$Q_O(t_i)$ = observed probability of failure or unreliability,
$Q_E(t_i)$ = expected probability of failure or unreliability.

**TABLE 3** – Parameter estimates obtained using the graphic and the proposed methods for the Example.

| Parameters | $\hat{p}$ | $\hat{\beta}_1$ | $\hat{\eta}_1$ | $\hat{\beta}_2$ | $\hat{\eta}_2$ | $D_{MAX}$ |
|---|---|---|---|---|---|---|
| **Graphic Estimates** | 0.3 | 0.6 | 39.4 | 2.3 | 233.5 | 0.075 |
| **Proposed Estimates** | 0.3 | 0.5 | 50.0 | 1.9 | 193.0 | 0.032 |

## 3. *CONCLUSIONS*

The proposed method, which combines the least-squares method with the Bayesian Method, makes full use of the information on the distributions' behavior of two subpopulations over the whole test duration and takes advantage of the simple parameter estimation for single-Weibull distributions. Therefore, it may yield more accurate parameter estimates. The following conclusions may be derived from this paper:

- The proposed method is more accurate than conventional methods. Its superiority is particularly significant for the small sample size case and a well-mixed population.
- The proposed method can be applied to the complete, censored, ungrouped and grouped samples. It always can find proper estimates for any data sample.
- The proposed method is easy to program since the closed forms for every computing step are given.
- The proposed method can be applied to other mixed distributions.

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