4

Lista 3

Cap. 4 – Variáveis Aleatórias Contínuas e Distribuições de Probabilidade

Ex. 1:

a) $P(1<X)=P(X>1)=\int_1^{\infty} f(x)dx=\int_1^{\infty} e^{-x}dx=\left[-e^{-x}\right]\Big|_1^{\infty}=-e^{-\infty}+e^{-1}\cong 0,368$

b) $P(1<X<2,5)=\int_1^{2,5} e^{-x}dx=\left[-e^{-x}\right]\Big|_1^{2,5}=-e^{-2,5}+e^{-1}\cong 0,286$

c) $P(X=3)=\int_3^{3} e^{-x}dx=\left[-e^{-x}\right]\Big|_3^{3}=-e^{-3}+e^{-3}=0$

d) $P(X<4)=\int_0^{4} e^{-x}dx=\left[-e^{-x}\right]\Big|_0^{4}=-e^{-4}+e^{0}=-0,018+1\cong 0,982$

e) $P(3\leq X)=P(X\geq 3)\int_3^{+\infty} e^{-x}dx=\left[-e^{-x}\right]\Big|_3^{+\infty}=-e^{-\infty}+e^{-3}\cong 0,050$

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Ex. 2:

a) $P(X<2{,}5)=\int_{-\infty}^{2,5} f(x)\,dx=\int_{2}^{2,5}(0{,}5x-1)\,dx=\left[0{,}5\frac{x^2}{2}-x\right]_2^{2,5}$

$$=\left[0{,}5\frac{2{,}5^2-2^2}{2}-(2{,}5-2)\right]=0{,}5\frac{2{,}25}{2}-0{,}5=0{,}0625$$

b) $P(X>3)=\int_{3}^{+\infty} f(x)\,dx=\int_{3}^{4}(0{,}5x-1)\,dx=\left[0{,}5\frac{x^2}{2}-x\right]_3^{4}=\left[0{,}5\frac{16-9}{2}-(4-3)\right]=0{,}75$

c) $P(2{,}5<X<3{,}5)=\int_{2,5}^{3,5}(0{,}5x-1)\,dx=\left[0{,}5\frac{3{,}5^2-2{,}5^2}{2}-(3{,}5-2{,}5)\right]=0{,}50$

d) $F(x)=P(X<x)=\int_{2}^{x}(0{,}5u-1)\,du=\left[0{,}5\frac{x^2-2^2}{2}-(x-2)\right]$

$$=\frac{x^2}{4}-x+1,\ \text{para } 2\le x\le 4$$

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Ex. 2 (cont.):

e) $\mu = E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_2^4 x(0,5x-1)\,dx = \left[0,5\frac{x^3}{3} - \frac{x^2}{2}\right]_2^4$

$$= \left[0,5\frac{4^3-2^3}{3} - \frac{4^2-2^2}{2}\right] = \frac{56}{6} - \frac{12}{2} = \frac{10}{3}$$

$$\sigma^2 = V(X) = E(X-\mu)^2 = E(X^2) - E(X)^2$$

$$= \int_{-\infty}^{+\infty} x^2 f(x)\,dx - \mu^2 = \int_2^4 x^2(0,5x-1)\,dx - \left(\frac{10}{3}\right)^2$$

$$= \left[0,5\frac{x^4}{4} - \frac{x^3}{3}\right]_2^4 - \frac{100}{9} = \left[\frac{4^4-2^4}{8} - \frac{4^3-2^3}{3}\right] - \frac{100}{9} = \frac{2}{9}$$

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Ex. 3:

a) $E(X) = 1/\lambda = 10\,\text{min} \Rightarrow \lambda = 0{,}10$ chamadas/ min

$$P(X < 5) = \int_0^5 f(x)\,dx = \int_0^5 \lambda e^{-\lambda x} dx = \left[-e^{-\lambda x}\right]_0^5 = \left[-e^{-0{,}1\times 5} + e^0\right] \cong 0{,}393$$

b) $P(5 < X < 15) = \int_5^{15} \lambda e^{-\lambda x} dx = \left[-e^{-\lambda x}\right]_5^{15} = \left[-e^{-0{,}1\times 15} + e^{-0{,}1\times 5}\right] \cong 0{,}383$

c) $P(X < x) = \int_0^x \lambda e^{-\lambda u} du = 0{,}9 \Rightarrow$

$$\int_0^x \lambda e^{-\lambda u} du = \left[-e^{-\lambda u}\right]_0^x = \left[-e^{-0{,}1x} + e^0\right] = 0{,}9 \Rightarrow$$

$$1 - 0{,}9 = e^{-0{,}1x} \Rightarrow \ln\left(e^{-0{,}1x}\right) = \ln(0{,}1) \Rightarrow x = -\ln(0{,}1)/0{,}1 \cong 23\,\text{min}$$

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Ex. 4:

a) $X \sim N(\mu = 5, \sigma = 0,2) \Rightarrow Z = \dfrac{X-\mu}{\sigma} \sim N(0,1)$

$$P(X > 5,5) = P\left(\frac{X-\mu}{\sigma} > \frac{5,5-5}{0,2}\right) = P(Z > 2,5)$$

$$P(Z > 2,5) = 1 - P(Z < 2,5) = 1 - \underbrace{\Phi(2,5)}_{\text{olhar na tabela}} \cong 1 - 0,994 = 0,006$$

b) $P(X < 4,5) + P(X > 5,5) = 1 - P(4,5 < X < 5,5) = 1 - P\left(\dfrac{4,5-5}{0,2} < Z < \dfrac{5,5-5}{0,2}\right)$

$$= 1 - \left[P(Z < 2,5) - P(Z < -2,5)\right] = 1 - \left\{P(Z < 2,5) - \left[1 - P(Z < 2,5)\right]\right\}$$

$$= 1 - \underbrace{\Phi(2,5)}_{\text{olhar na tabela}} + 1 - \underbrace{\Phi(2,5)}_{\text{olhar na tabela}} \cong 2 - 2 \times 0,994 = 0,012$$

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Ex. 4 (cont.):

c) $P(Z < z) = 0,90 \Rightarrow \underbrace{\Phi(z)}_{\text{olhar na tabela}} = 0,90 \Rightarrow z \cong 1,29$

$$z = \frac{x-\mu}{\sigma} \cong 1,29 \Rightarrow x \cong \mu + 1,29\sigma = 5 + 1,29 \times 0,2 = 5,258$$

$$P(X < 5,258) \cong 0,90$$