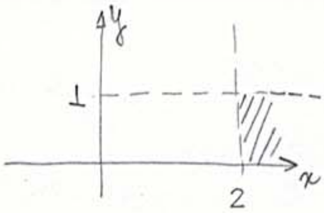
ex. 7 12



$f(x) = 8/x^3 \quad , \quad x > 2$

$g(y) = 2y \quad , \quad 0 < y < 1 .$

a) f.d.p. de $Z = XY$

$$k(v,z) = f\left[\underbrace{G_1(v,z)}_{x}, \underbrace{G_2(v,z)}_{y}\right] \, |J(v,w)|$$

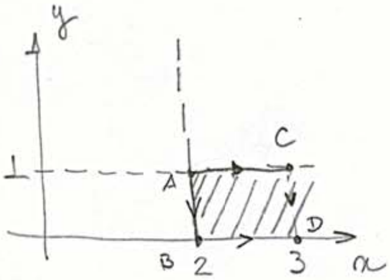
- $(x,y) \longmapsto (v,z)$

$z = H_1(x,y) = xy \quad \Rightarrow \quad x = G_1(v,z) = z/v$

$v = H_2(x,y) = y \quad \Rightarrow \quad y = G_2(v,z) = v$

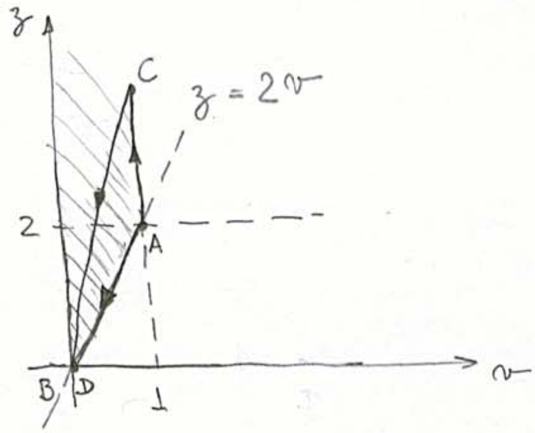
$$\therefore k(v,z) = \frac{8}{(z/v)^3} \cdot 2(v) \begin{vmatrix} \frac{\partial x}{\partial v} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial v} & \frac{\partial y}{\partial z} \end{vmatrix} = \frac{16 v^4}{z^3} \begin{vmatrix} -z/v^2 & 1/v \\ 1 & 0 \end{vmatrix}$$

$$k(v,z) = \frac{16 v^3}{z^3} \quad , \quad 0 \leq v \leq 1 , \quad z \geq 2v$$



$\Rightarrow$

$z = 2y$

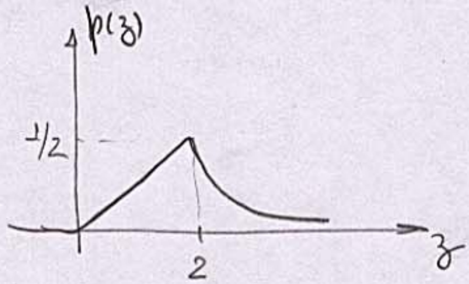


(x,y)	$\rightarrow$	(v,z)	
$(2,1)$		$(1,2)$	reta $z = 2v$
$(2,0)$	$\rightarrow$	$(0,0)$	
$(3,1)$	$\rightarrow$	$(1,3)$	reta $z = 3v$
$(3,0)$	$\rightarrow$	$(0,0)$	

reta $v = 1$

b) i) $$p(z) = \begin{cases} \displaystyle\int_0^{z/2} \frac{16v^3}{z^3}\, dv, & 0 < z < 2 \\[2ex] \displaystyle\int_0^{1} \frac{16v^3}{z^3}\, dv, & z > 2 \end{cases}$$

$$p(z) = \begin{cases} \displaystyle\frac{\cancel{16}^{\,4}}{z^3} \cdot \frac{v^4}{\cancel{4}_{\,1}}\Bigg|_0^{z/2} = \frac{4}{z^3}\,\frac{z^4}{2^4} = \frac{z}{4}, & 0 < z < 2 \\[2ex] \displaystyle\frac{\cancel{16}^{\,4}}{z^3} \cdot \frac{v^4}{\cancel{4}_{\,1}}\Bigg|_0^{1} = \frac{4}{z^3}, & z > 2 \end{cases}$$



$$E(Z) = \int_0^2 z\left(\frac{z}{4}\right)dz + \int_2^{+\infty} z\left(\frac{4}{z^3}\right)dz = \frac{1}{4}\,\frac{z^3}{3}\Bigg|_0^2 + 4\,\frac{z^{-1}}{(-1)}\Bigg|_2^{+\infty}$$

$$= \frac{1}{\cancel{4}_{\,1}}\,\frac{\cancel{8}^{\,2}}{3} + \left[\cancel{4\,\frac{(+\infty)^{-1}}{(-1)}}^{\,\to 0} - 4 \cdot \frac{2^{-1}}{(-1)}\right] = \frac{2}{3} + 2 = \frac{8}{3}$$