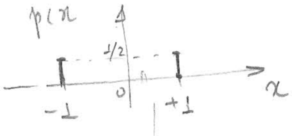
7.34) X

x_i	-1	$+1$
$p(x_i)$	$1/2$	$1/2$



a)

$$P\left[|X-E(X)| \geq k\sqrt{V(X)}\right] = ?$$

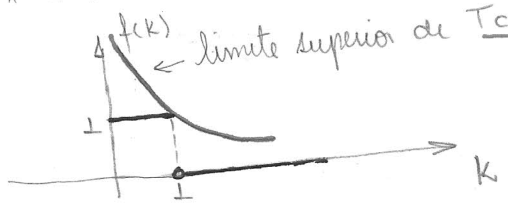
$$E(X) = \sum x_i\, p(x_i) = (-1)\,1/2 + (1)\,1/2 = 0$$
$$E(X^2) = \sum x_i^2 p(x_i) = (-1)^2 1/2 + (1)^2 1/2 = 1 \Rightarrow V(X) = E(X^2) - [E(X)]^2 = 1$$

$$P\left[|X-E(X)| \geq k\sqrt{V(X)}\right] = P\left[|X| \geq k\right] = f(k)$$

<u>Tchebycheff</u>:

$$P\left[|X-E(X)| \geq k\sqrt{V(X)}\right] \leq \frac{1}{k^2}$$

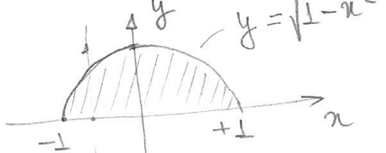
$f(k)$ ← limite superior de <u>Tchebycheff</u>, $1/k^2$



O limite realmente <u>funciona</u>

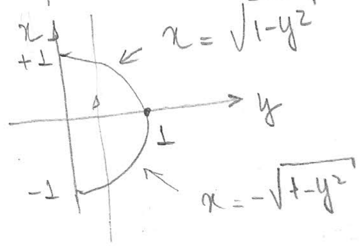
$f(k) = P[|X| \geq k]$

7.37) (X,Y) uniformente distribuída em $\{(x,y) \mid x^2+y^2 \leq 1,\ y \geq 0\}$



$$\rho_{xy} = ?$$

$$\rho_{xy} = \frac{E(XY) - E(X)E(Y)}{\sqrt{V(X)\,V(Y)}}$$



Determinação da conjunta $f(x,y)$

$$f(x,y) = \begin{cases} k, & (x,y) \mid x^2+y^2 \leq 1,\ y \geq 0 \\ 0, & c.c \end{cases}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\,dx\,dy = 1 \Rightarrow c \iint_{x^2+y^2 \leq 1} dx\,dy = 1$$

Podemos avaliar $\iint_{x^2+y^2\leq 1} dx\,dy$ por coordenadas polares, ou, mais simplesmente, notando que representa a área de meio círculo de raio unitário

$$\iint_{x^2+y^2 \leq 1} dx\,dy = \frac{\pi}{2}$$

Logo,

$$c\,\frac{\pi}{2} = 1 \Rightarrow c = \frac{2}{\pi}$$

Marginais:

$$g(x) = \int_0^{\sqrt{1-x^2}} \frac{2}{\pi}\,dy = \frac{2}{\pi}\sqrt{1-x^2}, \quad -1 \leq x \leq +1$$

$$h(y) = \int_{-\sqrt{1-y^2}}^{+\sqrt{1-y^2}} \frac{2}{\pi}\,dx = \frac{4}{\pi}\sqrt{1-y^2}, \quad 0 \leq y \leq 1$$

7.37 (continuação)

$$E(X) = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_{-1}^{+1} \frac{2}{\pi} x \sqrt{1-x^2}\, dx = \frac{2}{\pi}\left(-\frac{1}{3}\right)(1-x^2)^{\frac{3}{2}} \Big|_{-1}^{+1} = 0 \checkmark$$ ②

$$E(Y) = \int_{-\infty}^{+\infty} y f(y)\, dy = \int_{0}^{1} \frac{4}{\pi} y \sqrt{1-y^2}\, dy = \frac{4}{\pi}\left(-\frac{1}{3}\right)(1-y^2)^{3/2} \Big|_{0}^{1} = \frac{4}{3\pi} \checkmark$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f(x)\, dx = \int_{-1}^{+1} \frac{2}{\pi} x^2 \sqrt{1-x^2}\, dx = \frac{1}{4}$$

$$E(Y^2) = \int_{-\infty}^{+\infty} y^2 f(y)\, dy = \int_{0}^{1} \frac{4}{\pi} y^2 \sqrt{1-y^2}\, dy = \frac{1}{4}$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{1}{4}$$

$$V(Y) = E(Y^2) - [E(Y)]^2 = \frac{1}{4} - \left(\frac{4}{3\pi}\right) =$$

$$E(XY) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy\, f(x,y)\, dx\, dy = \frac{2}{\pi} \int_{-1}^{+1} x \int_{0}^{\sqrt{1-x^2}} y\, dy\, dx = \frac{2}{\pi} \int_{-1}^{+1} x \left[\frac{y}{2}\right]_{0}^{\sqrt{1-x^2}} dx$$

$$= \frac{2}{\pi} \int_{-1}^{+1} \frac{x}{2}(1-x^2)\, dx = \frac{1}{\pi}\left(\frac{x^2}{2} - \frac{x^4}{4}\right)\Big|_{-1}^{+1} = \frac{1}{\pi}\left[\left(\frac{1}{2}-\frac{1}{2}\right) - \left(\frac{1}{4}-\frac{1}{4}\right)\right] = 0$$