Exemplo 7.23

$X \sim \text{Unif}(10,30) \Rightarrow g(x) = 1/20, \ 10 < x < 30$ e $g(x) = 0$, c.c.

$Y \sim \text{Unif}(10,20) \Rightarrow h(y) = 1/10, \ 10 < y < 20$ e $h(y) = 0$, c.c.

$T \to$ lucro

$E(T) = ?$

$E[E(T|X)] = E(T)$, e é mais fácil

$$T = H(X,Y) = \begin{cases} 0{,}03Y & , \text{ se } Y < X, \\ 0{,}03X + 0{,}01(Y-X), & \text{ se } Y > X \end{cases} = \begin{cases} 0{,}03Y, & \text{se } Y < X \\ 0{,}01Y + 0{,}02X, & \text{se } Y > X \end{cases}$$

Da Eq 7.23:

$$E(T|X) = E(H(X,Y)|X) = \int_{-\infty}^{+\infty} H(X,Y) \cdot h(y|x) \, dy$$

Da Eq 6.7:

$$h(y|x) = \frac{f(x,y)}{g(x)}$$

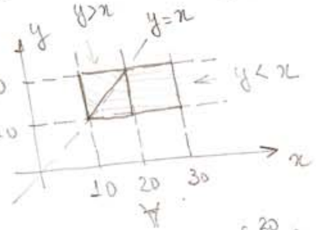
Da independência entre X e Y

$$f(x,y) = g(x) * h(y)$$

Logo:

$$E(T|X) = \int_{-\infty}^{+\infty} H(X,Y) \cdot \frac{\cancel{g(x)} \, h(y)}{\cancel{g(x)}} \, dy$$

Como:



$$E(T|X) = \begin{cases} \int_{10}^{x} 0{,}03y \, h(y) \, dy + \int_{x}^{20} [0{,}01y + 0{,}02x] \, h(y) \, dy, & 10 < x < 20 \\ \int_{10}^{20} 0{,}03y \, h(y) \, dy & , \ 20 < x < 30 \end{cases}$$

$h(y) = 1/10$

$$\therefore E(T|X) = \begin{cases} \frac{1}{10} 0{,}03 \frac{y^2}{2} \Big|_{10}^{x} + \frac{1}{10} \left[0{,}01 \frac{y^2}{2} + 0{,}02xy \right]_{x}^{20} & , \ 10 < x < 20, \\ \frac{1}{10} \left[0{,}03 \frac{y^2}{2} \right]_{10}^{20} & , \ 20 < x < 30, \end{cases}$$

$$E(T|X) = \begin{cases} \frac{1}{10}\left[(0{,}015\,x^2 - 1{,}5) + (2 - 0{,}005\,x^2) + (0{,}4x - 0{,}02x^2)\right], & 10 < x < 20, \\ \frac{1}{10}\left[0{,}03\cdot\left(\frac{400}{2} - \frac{100}{2}\right)\right], & 20 < x < 30, \end{cases}$$

$$E(T|X) = \begin{cases} 0{,}05 + 0{,}04x - 0{,}001\,x^2, & \text{se } 10 < x < 20, \\ 0{,}45, & \text{se } 20 < x < 30. \end{cases}$$

Da Eq 7.6

$$E[\underbrace{E(T|X)}_{H(X)}] = \int_{-\infty}^{+\infty} H(x)\cdot g(x)\,dx$$

Como $g(x) = 1/20$, $10 < x < 30$, pois $X \sim \text{Unif}(10, 30)$

$$E[E(T|X)] = \int_{10}^{30} E(T|X)\cdot\frac{1}{20}\cdot dx$$

Pela definição de $E(T|X)$

$$E[E(T|X)] = \frac{1}{20}\int_{10}^{20}(0{,}05 + 0{,}04x - 0{,}001x^2)\,dx + \frac{1}{20}\int 0{,}45\,dx = \$0{,}43.$$