On the Output of M/G/C/C State Dependent Queueing Networks

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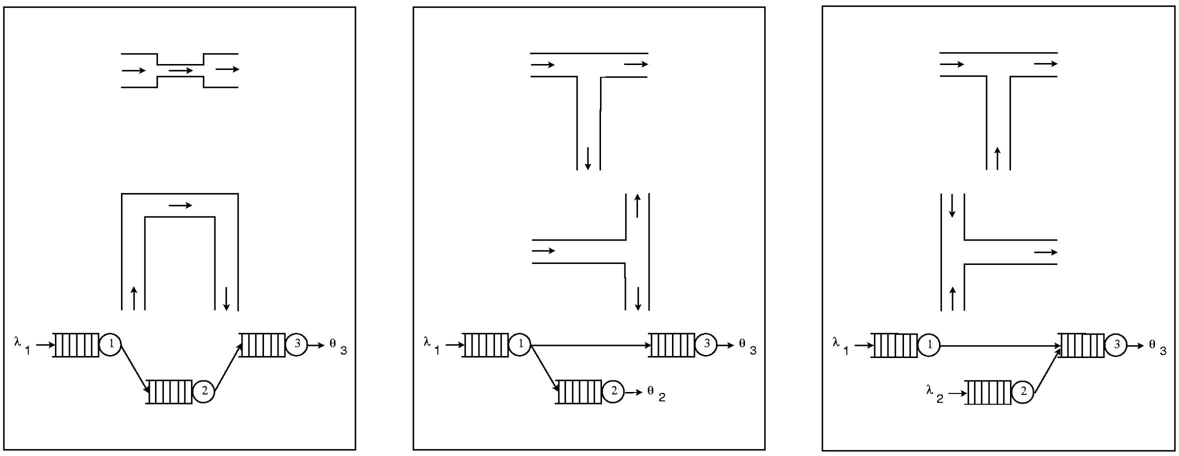
Abstract

Networks of M/G/C/C state-dependent queueing are important tools to model congestion. Since the service times are a complex function of the number of users in the system, the stochastic process beneath the departure process is not well known even under Markovian arrivals. By using a discrete-event simulation approach, this paper provides some insights into the output process of such systems evidencing that an exponential interdeparture time may be a reasonable assumption. The result has important implications to practical network of queues configured in tandem, merge, split, and general topologies.
Key words: Queueing Systems, Finite Capacity; State Dependent; Discret-event.

1. Introduction

An useful tool to model some special real world systems subject to congestion are M/G/C/C state dependent queueing networks. Following Kendall's notation, these are systems under Markovian arrivals, general state dependent service rates, C servers in parallel, and a total capacity of C users. Examples of systems successfully modeled as M/G/C/C state dependent queueing networks include general topological network design problems (MACGREGOR SMITH, 1996; MITCHELL & MACGREGOR SMITH, 2001), industrial engineering problems (MACGREGOR SMITH & DASKALAKI, 1988; MACGREGOR SMITH & CHIKHALE, 1995), vehicular traffic flows (MACGREGOR SMITH, 1994; JAIN & MACGREGOR SMITH, 1996), and pedestrian flow networks (YUHASKI & MACGREGOR SMITH, 1989; BAKULI & MACGREGOR SMITH, 1996).



a) series b) split c) merge

Figure 1: Corridor topologies.

Let us take pedestrian flow networks as an example. The modeling of configurations as simple as those presented in Figure 1, may require several G/G/C/C queues, i.e., with arrivals also general, configured in a tandem, split, or merge topologies. In other words, practical situations would call for complex configurations and since the intermediate departure processes is unknown, perhaps the M/G/C/C model might not be as useful as it seems to be.

This paper is inspired by the classical work of Burke (1956), developed for M/M/1 systems and its main purpose is to verify whether or not M/G/C/C state dependent queueing networks are a reasonable choice to modeling configurations similar to those shown in Figure 1. In other words, we intend to provide experimental evidences in favor of Conjecture 1.

Conjecture 1 *The departure process from an $M/G/C/C$ state dependent queue is Poisson.*

The paper is organized as follows. Section 2 presents the congestion model and Section 3, the simulation results. Section 4 concludes the paper with final remarks and future direction for research in the area.

2. Congestion model

Although the congestion model to be presented here is applied to pedestrian systems, the methodology is general. Examples for other areas can be found in the papers by Thumsi & MacGregor Smith (1998) and Jain & MacGregor Smith (1997).

A corridor connecting two locations can be viewed as a service mechanism for its occupants. The number of servers in parallel is equal to the node capacity which also represents the total number of users simultaneously allowed in the system, that is:

$$c = \lfloor 5 * L * W \rfloor, \tag{1}$$

in which L is the nodal length and W, its width in meters. Notice that 5 ped/m^2 represents the maximum pedestrian density, in accordance to Tregenza (1976).

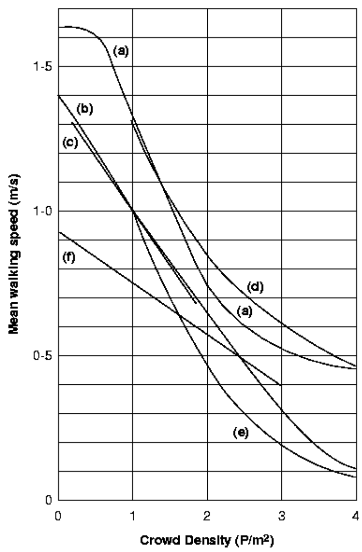


Figure 2: Average walking speed (TREGENZA, 1976).

Also in accordance to Tregenza, (1976), the average speed that a user crosses a traffic link depends on several factors but mainly this speed is a function of the number of occupants. Figure 2 (TREGENZA, 1976) presents experimental curves, (a) through (f), relating the walking speed of a pedestrian to the pedestrian density.

Based on the above remarks, Yuhaski & MacGregor Smith (1989) developed the following linear and exponential congestion models for the average pedestrian walking speed in traffic links:

$$f(n) = \frac{V_n}{V_1} = \frac{(c+1-n)}{\text{c}} \tag{1}$$

and

$$f(n) = \frac{V_n}{V_1} = \exp\left[-\left(\frac{n-1}{\beta}\right)^{\gamma}\right], \tag{2}$$

in which γ and β are shape and scale parameters for the exponential model, given by:

$$\gamma = \ln\left[\frac{\ln(V_a / V_1)}{\ln(V_b / V_1)}\right] / \ln\left(\frac{a-1}{b-1}\right) \tag{3}$$

and

$$\beta = \frac{a-1}{[\ln(V_1 / V_a)]^{\frac{1}{\gamma}}} = \frac{b-1}{[\ln(V_1 / V_b)]^{\frac{1}{\gamma}}} \tag{4}$$

in which V_1 is the average speed for lone occupant, assumed 1.5 m/s, V_a is the average speed in m/s when the density is 2 ped/m^2, $a=2LW$, V_b is the average speed when crowd density is 4 ped/m^2, and b=4LW.

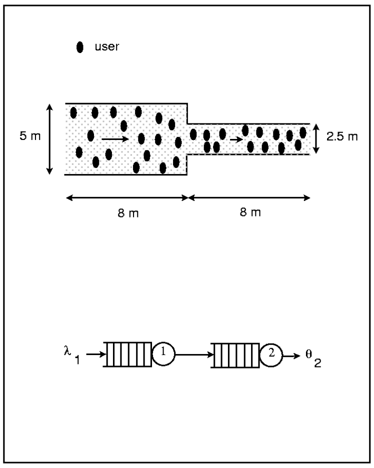


Figure 3: System under analysis.

Also pointed out by Yuhaski & MacGregor Smith (1989) is that the exponential model could be adjusted based on 3 points averaged over the 6 curves in Figure 2. Other possibilities also exists, e.g., non-linear regression or piece-wise linear approximations, but the results would not differ significantly.

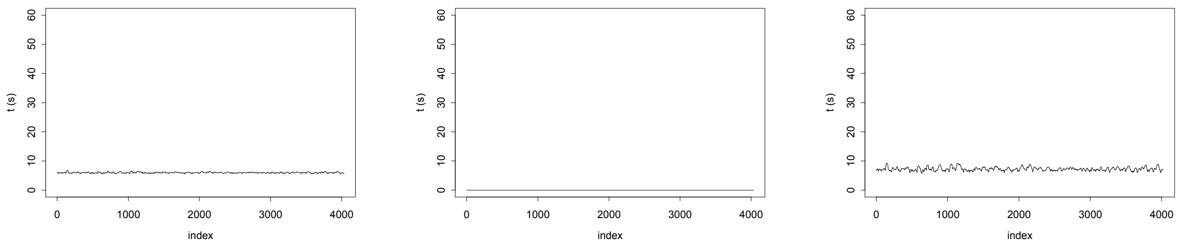
Throughout this paper, an exponential model is assumed, with V_a=0.64 m/s and V_b=0.25 m/s. A discrete-event digital simulation model (CRUZ ET AL., 2002) has confirmed the accuracy of these settings. As a final remark, Cheah & MacGregor Smith (1994) successfully extended the exponential model to represent bi-directional and multi-directional pedestrian flows by using slightly different values for V_a and V_b.

3. Simulation study

In order to study the output of M/G/C/C state dependent queueing networks, a discrete-event digital simulation model, coded in C++, developed by Cruz et al. (2001), is used. For a detailed discussion about the ideas behind the model as well as a presentation of validating results and an interesting example application related to evacuation networks, the reader may refer to the paper by Cruz et al. (2001), available through the web. The code is also available on request.

The system under analysis can be seen in Figure 3. It is composed by two corridors, 8 x 5 m^2 and 8 x 2.5 m^2 and is modeled as a two-node M/G/C/C state dependent queueing networks in tandem. Someone may argue that the problem is rather simple but as we shall see it is general enough for our purposes. Notice that other modeling alternatives exist. However, the simulation model is robust enough to produce similar results to those that will be presented shortly.

We remark that the bottleneck in the end node is the most difficult case to be analyzed because of the strong dependency it may cause among the two queues, depending on the arrival rate. Notice that, under a heavy traffic and the bottleneck being otherwise in the front node, part of the incoming flow would be just forced to balk, consequently decoupling the two queues. Thus, one would have a Jackson network whose queues could be analyzed independently, given that the output of an M/G/C/C queue is well-known to be Poisson (CHEAH & MACGREGOR SMITH, 1994). Soon, we shall see experimental evidence on these two remarks.

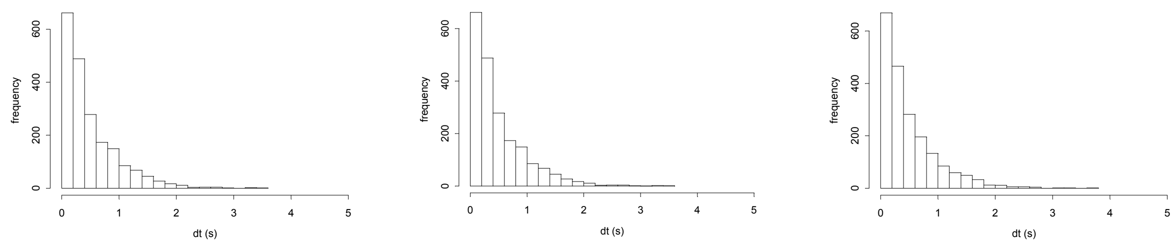


a1) being served   a2) blocked after service   b) end node

Figure 4: Service times along the time (λ=2).

Figure 4 shows (a1) the service times in the front node, (a2) the waiting time for space at the end node, and (b) the service time in the end node. We assumed an arrival rate of 2.0 ped/s and 2,000 seconds of total time simulated. Notice that an egress from the front node may be subjected to blocking after service but in this case there will be none, since this arrival rate is too light and easily handled by the end node without any blocking. This can be easily seen in Figure 4-a2.

In this case, the two queues are in fact decoupled and it is well known that under such circumstances the departure process in each of them must be Poisson (CHEAH & MACGREGOR SMITH, 1994). Indeed, one can see in Figure 5 and Table 1 how accurate were the interdeparture times produced by the simulation model. The histograms are visually close to a exponential distribution and the data have roughly same mean and standard deviation.



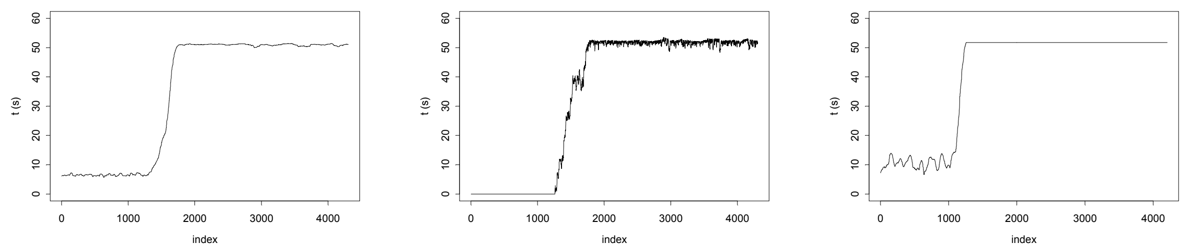
a1) being served a2) blocked after service b) end node

Figure 5: Interdeparture time histograms (λ=2).

Tabela 1: Descriptive statistics for the interdeparture time.

λ	node	Min.	Q1	Median	Mean	Std.Dev.	Q3	Max.
2	1 (in service)	0.000122	0.1433	0.3402	0.4895	0.4748	0.6858	3.515
	1 (blocked)	0.000122	0.1433	0.3402	0.4895	0.4748	0.6858	3.515
	2	0.000122	0.1468	0.3368	0.4893	0.476	0.6861	3.660
3	1 (in service)	0.000732	0.1511	0.3586	0.5174	0.4834	0.7376	3.249
	1 (blocked)	0.002136	0.1439	0.3438	0.5176	0.4872	0.793	2.146
	2	0.002136	0.1558	0.3555	0.5173	0.4862	0.7811	2.146

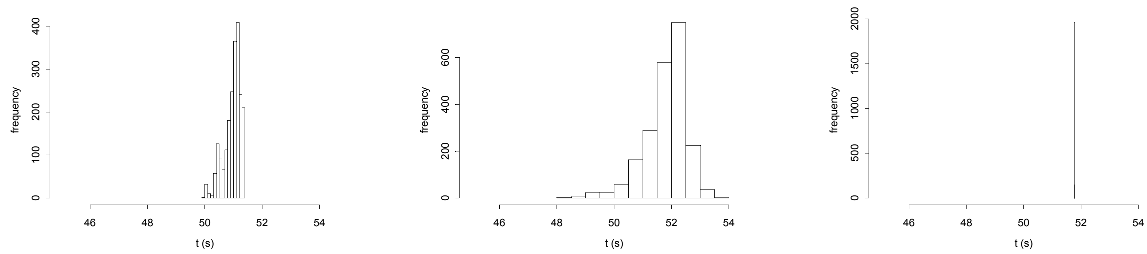
A more interesting case is presented in Figure 6. Here we assume na arrival rate of 3.0 ped/s and 2,000 seconds of total time simulated. Notice that there will be many users blocked, since now the arrival rate is too high to be handled by the end node without any blocking, as seen in Figure 6-a2.



a1) being served a2) blocked after service b) end node

Figure 6: Service times along the time (λ=3).

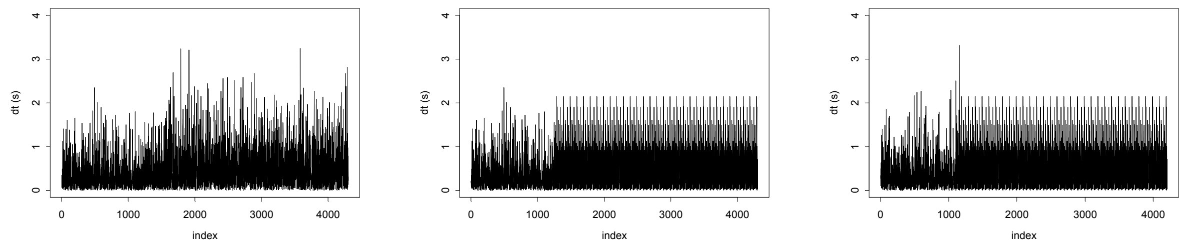
The first important fact to be observed here is that it is necessary to discard a warm-up period before making any steady-state analysis. So, ignoring the first half of the 2,000 seconds simulated, we can drawn histograms for the service time as seen in Figure 7. Notice that the system is so overloaded that the service time is almost 10 times larger than the lone occupant service time (8.0/1.5=5.33). Also, notice how differently the service times histograms are. Indeed, the downstream node has a virtually deterministic service time. Here, we can convince ourselves that a more flexible analytical model than simply an M/M must be used if more accuracy in modeling congestion is needed.



A1) being served a2) blocked after service b) end node

Figure 7: Service time histograms (λ=3).

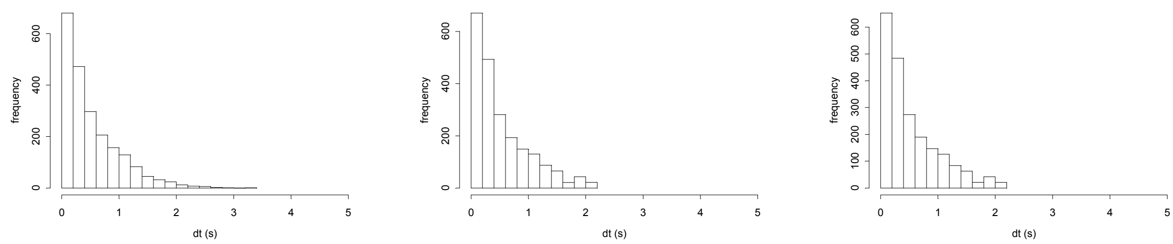
Studying the interdeparture times for this system, we can drawn the graphs presented in Figure 8.



A1) being served a2) blocked after service b) end node

Figure 8: Interdeparture times along the time (λ=3).

Again, discharging the first half of the observations in order to avoid transient perturbations, we can obtain the histograms presented in Figure 9 and find out how surprisingly close the departure processes at all nodes are from an exponential distribution. The descriptive statistics presented in Table 1 reinforce Conjecture 1 even more.



A1) being served a2) blocked after service b) end node

Figure 9: Interdeparture time histograms (λ=3).

4. Summary and conclusion

We stressed the importance of M/G/C/C state depend congestion models for modeling real world concern problems and mentioned some recently published research results on the subject. Since practical systems may need a complex arrange of M/G/C/C queues, configured in arbitrary topologies, including series, merges, and splits, we focused on studying the departure process for such systems. Simulation results indicated that the interdeparture times may be fairly assumed as exponentially distributed. However, a complete understanding of the stochastic process involved still resists analytical analysis and might be the focus of future research efforts.

Acknowledgments

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