Solving a Local Access Urban Telecommunication Network Design Problem

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Abstract — This paper deals with a multi-level network optimization (MLNO) problem that raises optimization aspects of dimensioning, topological design, and facility location. In this sense, the model can be applied in network planing to explore design aspects in a modeling approach that integrates several hierarchical levels. We apply the model to a local access urban telecommunication network design problem and a branch-and-bound algorithm is applied to solve it. Computational results are provided, illustrating the effectiveness of this technique.

Keywords — Network design problems, multi-level networks, telecommunication networks, location problems.

1 Introduction

Network planning must satisfy the expected demand for new services, upgrading, improvements on the existing network, quality of service (QoS), and performance at minimum cost. The aim of this paper is to explore the hierarchical organization of each network and to illustrate the use of a integrated network model as a decision support system. In this context, we have focused solutions for basic urban mapping data capture and data analysis using a Geographic Information System (GIS) and network optimization systems (Mateus, Cruz and Luna, 1994; Mateus, Pádua and Luna, 1996). We apply the multi-level network optimization (MLNO) problem (Cruz, Smith and Mateus, 1999) on a local access urban telecommunication network design problem (Luna, Ziviani and Cabral, 1987; Balakrishnan, Magnanti, Shulman and Wong, 1991), that uses both optical fiber and copper cable links.

The MLNO problem is defined on a multi-weighted digraph $\mathcal{D} = (N, A)$, where N is the set of nodes and A is the set of arcs. Figure 1 shows a m-level network example containing candidate supply nodes, demand nodes, and transshipment nodes at each level. The objective is to determine an optimum combination of supply nodes and arcs to provide the required flow type to all demand nodes respecting rules of flow conservation.

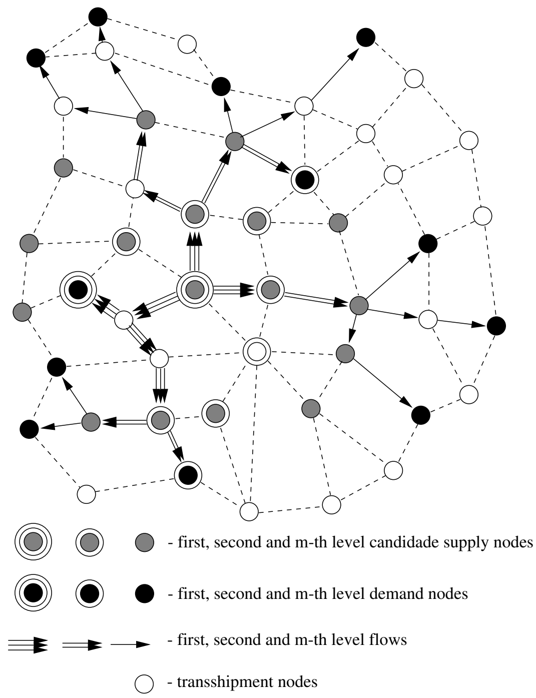

Figure 1: The MLNO Problem

The outline of the paper is as follows. In section 2, we present a mathematical programming formulation for the MLNO problem. In section 3, a branch-and-bound algorithm for the problem is presented briefly. Computational results are shown in Section 4, for a "real" network. We solve this real world sample problem to illustrate the effec-

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tiveness of our solution method. We conclude the work in Section 5, presenting and discussing some open questions.

2 Formulation

2.1 Assumptions

In formulating the MLNO problem, we made some assumptions concerning the settings which are made explicit below:

1. The arcs have cost parameters that include a non-negative fixed cost of using the arc and a non-negative cost per-unit of flow. There is a discontinuity in the zero flow values, so the total cost is a nonlinear function of the amount of flow.
2. The supply capacity of the first-level candidate supply nodes equals the sum of all demands in all levels.
3. The candidate supply nodes of the other levels are really "transformation" nodes. They receive flows from one level and convert them to another in a 1:1 ratio.

   These assumptions are consistent with what happens in practice in a two-level telecommunication network. A copper cable service user eventually has to be "transformed" first into an optical fiber service user before getting connected to another similar copper cable service user.

   The ratio 1:1 keeps the model simpler and does not make it less powerful. It would be possible to model whatever ratio by convenient adjustments on the flow scales. The electrical engineers are used to do so in studying electric power systems referring all voltages to one side of the transformers.
4. There is a cost for transforming flows from one level to another. We model here possible hardwares that must be present to interconnect the different networks.

2.2 Notation

We now define the notation used:

- m - number of levels;
- R^l - set of l-th level candidate supply nodes;
- D^l - set of l-th level demand nodes.
- d_i - l-th level demand node $i \in D^l$;
- T^l - set of l-th level transshipment nodes, defined as follows: $T^l = N \setminus (R^l \cup D^l \cup R^{l+1})$ for $l = 1, 2, \ldots, (m-1)$, and $T^m = N \setminus (R^m \cup D^m)$;
- c^l_{ij} - non-negative per-unit cost for l-th level flow on arc $(i,j) \in A$;
- x^l_{ij} - l-th level flow through arc $(i,j) \in A$;
- f^l_{ij} - non-negative fixed cost for using arc $(i,j) \in A$ to support l-th level flow;
- y^l_{ij} - boolean variable which assumes the value 1 or 0 depending on whether or not the arc (i,j) is being used to support l-th level flow;
- f_i - non-negative allocation cost for the l-th level candidate supply node $i \in R^l$;
- z_i - boolean variable which is set to 1 or 0 depending on whether or not the node $i \in R^l$ is being selected to provide l-th level flow;
- M^l - capacity on all arcs in the l-th level, but relaxed in this paper and considered a *big* enough number, *i.e.* $M^l = \sum_{L=l}^{m} \sum_{i \in D^L} d_i$;
- s^l - capacity on all l-th level candidate supply nodes, but also relaxed in this paper, *i.e.* $s^l = M^l$;
- $\delta^+(i)$ - set $\{j | (i,j) \in A\}$;
- $\delta^-(i)$ - set $\{j | (j,i) \in A\}$.

2.3 Formulation

The mathematical programming formulation describing the MLNO problem is presented as a flow-based mixed-integer programming (MIP) model:

Model (M)**:**

$$\min \sum_{l=1}^{m} \left[\sum_{(i,j) \in A} \left(c^l_{ij} x^l_{ij} + f^l_{ij} y^l_{ij} \right) + \sum_{i \in R^l} f_i z_i \right], \quad (1)$$

s.t.:

$$\sum_{j \in \delta^+(i)} x^l_{ij} - \sum_{j \in \delta^-(i)} x^l_{ji} = -\left(\sum_{j \in \delta^+(i)} x^{l-1}_{ij} - \sum_{j \in \delta^-(i)} x^{l-1}_{ji} \right), \quad \forall \ i \in R^l, \ l=2,3,\ldots,m, \quad (2)$$

$$\sum_{j \in \delta^+(i)} x^l_{ij} - \sum_{j \in \delta^-(i)} x^l_{ji} = 0, \quad \forall \ i \in T^l, \ l=1,2,\ldots,m, \quad (3)$$

$$\sum_{j \in \delta^+(i)} x^l_{ij} - \sum_{j \in \delta^-(i)} x^l_{ji} = -d_i, \quad \forall \ i \in D^l, \ l=1,2,\ldots,m, \quad (4)$$

$$\sum_{j \in \delta^+(i)} x^l_{ij} - \sum_{j \in \delta^-(i)} x^l_{ji} \le s^l z_i, \quad \forall \ i \in R^l, \ l=1,2,\ldots,m, \quad (5)$$

$$x^l_{ij} \le M^l y^l_{ij}, \quad \forall \ (i,j) \in A, \ l=1,2,\ldots,m, \quad (6)$$

$$x^l_{ij} \ge 0, \quad \forall \ (i,j) \in A, \ l=1,2,\ldots,m, \quad (7)$$

$$y_{ij}^l \in \{0,1\}, \quad \forall \quad \begin{matrix}(i,j) \in A,\\ {\scriptstyle l=1,2,\ldots,m,}\end{matrix} \quad (8)$$

$$z_i \in \{0,1\}, \quad \forall \quad \begin{matrix}i \in R^l,\\ {\scriptstyle l=1,2,\ldots,m.}\end{matrix} \quad (9)$$

The objective function (1) minimizes three terms: (i) the first accounts for the total flow's variable cost for all flow types, (ii) the second accounts for the fixed cost associated with the use of the arcs (the overhead cost), and (iii) the last considers the total cost resulting from the use of the supplying nodes.

Constraints (2) ensure the network flow conservation between adjacent levels at each supply candidate node. Constraints (3) and (4) are the usual network flow conservation equalities at each transshipment node and demand node. For example, from the point of view of level 1 all nodes $i \in N \setminus (R^1 \cup D^1 \cup R^2)$ are transshipment nodes (see Figure 1). Constraints (5) ensure there is no flow transformation in a candidate supply node if it is not selected, and constraints (6) express the fact that the flow through an arc must be zero if this arc is not included in the design.

3 Algorithm

An exact approach to solve any $\mathcal{NP}$-hard optimization problem normally is to implicitly enumerate all solutions. Branch-and-bound is a well-known technique largely applied to many similar problems. It can be tailored in order to get efficient solutions quickly. We propose implementations based on the branch-and-bound algorithm depicted in Figure 2. In that description, U_{BEST} is the global upper bound and Γ is a list of unexplored problems $(M)^i$, each of which is of the form $Z_M^i = \min\{\mathbf{cx\ s.t.:\ x} \in S^i\}$, where $S^i \subseteq S$ and S is the set of feasible solutions. Associated with each problem in Γ are a lower bound $L^i \leq Z_M^i$ and an upper bound $U^i \geq Z_M^i$. For memory economy purposes, the search rule applied was *last-in-first-out* which yields a *depth-first* search strategy.

```
algorithm Branch-and-Bound
    U_BEST ← +∞
    Γ ← {(M)^0}
    while Γ ≠ ∅ do
            /* search rule */
        select and delete a problem (M)^i from Γ
            /* bound rule */
        Compute_Lower_and_Upper_Bounds(L^i,U^i)
        update U_BEST
            /* branch rule */
        if L^i < U_BEST and (M)^i is not a leaf then
            Γ ← Γ ∪ {(M)^{2i+1}} ∪ {(M)^{2i+2}}
        end if
    end while
end algorithm
```

Figure 2: Branch-and-Bound Algorithm

3.1 Lower and Upper Bound Computations

An obvious way to compute lower bounds for MIP problems in general is through a linear programming relaxation and it could be applied to the MLNO problem. However, we will make use of the Lagrangean relaxation, which is a well know technique, used commonly coupled with subgradient optimization procedures (Fisher, 1981). The method idea is to drop complicating constraints, say constraints (5) and (6), by means of a Lagrangean multiplier vector. This relaxation is a lower bound for the original problem. The heuristic method used here to compute upper bounds is based on the solution of the Lagrangean relaxation. Basically, the method uses the Lagrangean relaxation solution (that is, in which constraints (5) and (6) where dropped) and compute the solution overhead, considering now that all constraints are satisfied. The interested reader may find more details in the literature (Cruz, Mateus and Smith, 1996).

4 Solving a "Real" Case

We shall now solve a realistic case. The problem presented arose at TELEMIG, the telephonic company of *Minas Gerais* State, Brazil. This is the problem of planning a local access urban telecommunication network (Luna et al., 1987), that uses both optical fiber and copper cable links. In this application, first and second levels are meant to represent both different transmission media, optical fibers and copper cables. The fixed costs associated with the transformation nodes (second-level candidate supply nodes) mean the cost of the hardware necessary to convert the optical signal into the electrical signal and vice-versa. The fixed costs in arcs usually represent the infrastructure costs, which in some sense are independent of the amount of flows the arc carries. In this context, the flow costs are obvious. We assume that the supply node in the first-level is able to provide as much flow as it is required. In other words, we assume that its capacity equals the sum of all demands.

Table 1: "Real" Case Settings

m	$=$	2
R^1	$=$	$\{1\}$
D^1	$=$	$\emptyset$
R^2	$=$	$N \setminus (R^1 \cup D^1 \cup R^2)$
	$=$	$\{2,\ldots,21,23,24,26,\ldots,33,37,40,\ldots,42\}$
D^2	$=$	$\{22,25,34,35,36,38,39,43\}$
d^i	$=$	$1,\ \forall i \in D^2$
T^1	$=$	$N \setminus (R^1 \cup D^1 \cup R^2) = \emptyset$
T^2	$=$	$N \setminus (R^2 \cup D^2) = \emptyset$

The graph in Figure 3 represents the urban topology of *Monlevade*, a medium sized town in *Minas Gerais* State. As said previously, the arcs repre-

Table 2: Basic Arc Costs Ω_{ij} for the Sample Problem

(i,j)	Ω_{ij}	(i,j)	Ω_{ij}	(i,j)	Ω_{ij}	(i,j)	Ω_{ij}
1 , 2	130	12, 5	85	18,19	40	36,35	315
3 , 2	50	13, 6	100	18,17	105	36,37	120
3 , 4	65	14, 7	110	17, 2	50	37,38	290
4 , 5	55	15, 8	130	2 ,10	50	38,39	160
5 , 6	65	19,10	100	18, 1	130	39,40	140
6 , 7	55	19,20	70	26,20	90	40,41	200
7 , 8	60	20,21	60	26,27	60	41,42	180
8 , 9	75	21,22	65	27,21	70	42,37	235
9 ,16	125	22,23	60	27,22	100	41,38	141
16,15	70	23,24	60	1 ,34	60	42,36	340
15,14	60	24,25	80	1 ,33	150	33,34	150
14,13	60	16,25	100	32, 3	55	18,26	130
13,12	60	20,11	100	28,32	95	33,43	150
12,11	55	21,12	95	32,31	50	43,30	50
11,10	70	22,13	100	29,31	125	30,29	60
10, 3	55	23,14	105	34,36	215	29,28	60
11, 4	70	24,15	100	35,34	230	28, 2	80

Table 3: Results for the Sample Problems

Case	$\frac{f^1_{ij}}{\Omega_{ij}}$	$\frac{c^1_{ij}}{\Omega_{ij}}$	$\frac{f^2_{ij}}{\Omega_{ij}}$	$\frac{c^2_{ij}}{\Omega_{ij}}$	f_i	U_{opt}	$\{i \mid z_i = 1\}$	Paths*
I	2	20	1	10	1	59,763	$\{1, 18, 33\}$	$1 \Rightarrow 18 \to 19 \to 20 \to 21 \to 22 \to 23 \to 24 \to 25$
								$1 \Rightarrow 18 \to 19 \to 20 \to 21 \to 22$
								$1 \Rightarrow 33 \to 34 \to 36 \to 37 \to 38 \to 39$
								$1 \Rightarrow 33 \to 34 \to 36 \to 37 \to 38$
								$1 \Rightarrow 33 \to 34 \to 36$
								$1 \Rightarrow 33 \to 34 \to 35$
								$1 \Rightarrow 33 \to 34$
								$1 \Rightarrow 33 \to 43$
II	1	10	2	20	1	61,356	$\{1, 24, 21, 30, 33, 37\}$	$1 \Rightarrow 18 \Rightarrow 19 \Rightarrow 20 \Rightarrow 21 \Rightarrow 22 \Rightarrow 23 \Rightarrow 24 \to 25$
								$1 \Rightarrow 18 \Rightarrow 19 \Rightarrow 20 \Rightarrow 21 \to 22$
								$1 \Rightarrow 2 \Rightarrow 28 \Rightarrow 29 \Rightarrow 30 \to 43$
								$1 \Rightarrow 33 \to 34 \to 35$
								$1 \Rightarrow 33 \to 34$
								$1 \Rightarrow 34 \Rightarrow 36 \Rightarrow 37 \to 38 \to 39$
								$1 \Rightarrow 34 \Rightarrow 36 \Rightarrow 37 \to 38$
								$1 \Rightarrow 34 \Rightarrow 36 \Rightarrow 37 \to 36$

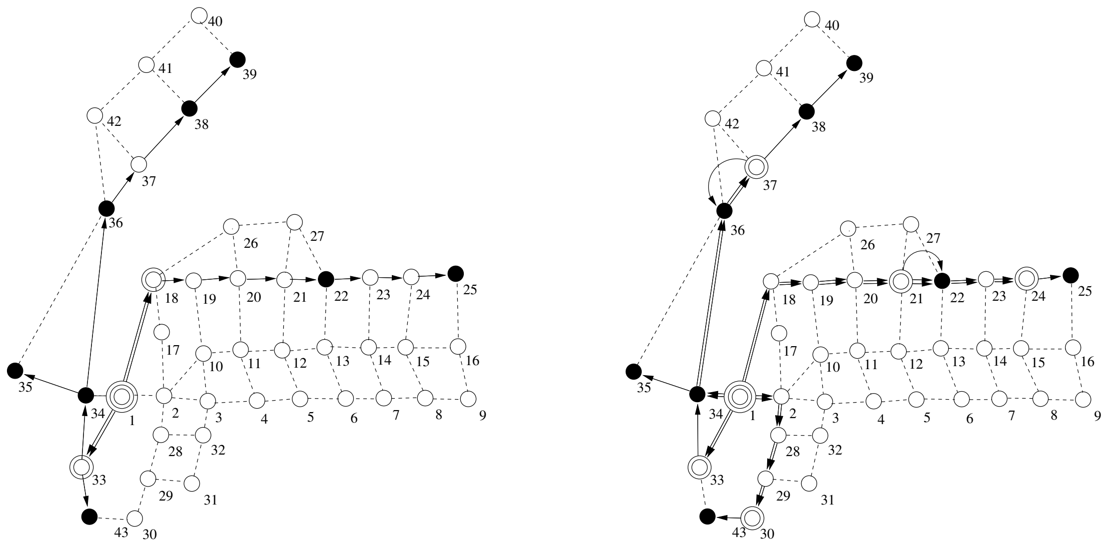
* $\Rightarrow$ first-level flow, $\to$ second-level flow

sent main streets and nodes represent concentrators of telephonic pairs or crosses. There is a total of 43 nodes, 68 edges (136 arcs) and 8 second-level demand nodes, considered *unitary* in this application example, although in the real world, the demand nodes usually concentrate 300 household subscribers or more. The first-level supplier is node 1 and the second-level candidate supply nodes are all remaining nodes. The MLNO problem settings are presented in Table 1.

Table 2 shows the arc distances Ω_{ij} in meters. We solved two cases. The arc costs used (fictitious) are proportional to the arc distances, as they are in practice, and are shown in Table 3. Table 3 and Figure 3 represent the optimal solutions found. Case I shows what would happen in a scenario in which the second-level media costs were lower than first-level media costs. We see a small number of concentrators and first-level arcs. As a matter of fact, the optimum solution would be to contract the first-level network as much as possible, avoiding the use of first-level flows. On the other hand, case II represents a more practical scenario. If we had first-level facility costs lower than second-level ones, we would tend to increase the size of the first-level network. That is exactly what happens nowadays. Because of a continuous reduction of their costs, we are watching an increasing spread of high speed networks.

4.1 Notes on Algorithm Performance

At the current stage, our implementations run considerably *slower* than a commercial software such as CPLEX (CPLEX Optimization, Inc., 1993). The



a) Case I   b) Case II

Figure 3: Results for the Sample Example

$U_{\mathrm{BEST}} \leftarrow +\infty$
$\Gamma \leftarrow \{(M)^0\}$
while *there is work* **do**
  if $\Gamma \neq \emptyset$ **then**
    select a problem $(M)^i$ *from* Γ
    send $(M)^i$ *and* U_{BEST} *to worker*
  end if
  if *there is an answer* **then**
    receive U'_{BEST} *and* Γ' *from worker*
    update U_{BEST}
    $\Gamma \leftarrow \Gamma \cup \Gamma'$
  end if
end while
terminate workers

a) Controller Algorithm

$U'_{\mathrm{BEST}} \leftarrow +\infty$
while *not terminate* **do**
  receive problem $(M)^i$ *and* U_{BEST}
  Compute_Lower_and_Upper_Bounds(L^i,U^i)
  update U'_{BEST}
  if $L^i < U'_{\mathrm{BEST}}$ **and** $(M)^i$ *is not a leaf* **then**
    $\Gamma' \leftarrow \{(M)^{2i+1}\} \cup \{(M)^{2i+2}\}$
  else
    $\Gamma' \leftarrow \emptyset$
  end if
  send U'_{BEST} *and* Γ' *to controller*
end while

b) Worker Algorithm

Figure 4: Parallel Branch-and-Bound Algorithm

approach presented here is not the fastest available but it has the advantage of being "open" and easily adaptable for other platforms, including parallel computers. The idea of reducing the computational time of branch-and-bound algorithms by means of parallelism is very promising (Laursen, 1993; Gendron and Crainic, 1994). We could use a parallel version of the algorithm presented in Figure 2 which will be based on a *controller-worker* approach. The *controller* is responsible for the general initializations, creation of the *worker* processes, and their coordination. The high level algorithm for this version is presented in Figure 4. In such a version, the *controller* manages the list Γ of unexplored problems $(M)^i$ but the task of expanding the problems is attributed to the *workers*, *i.e.* the *worker* processes are responsible for computing the bounds and generating the respective children.

In other words, the *controller* keeps itself in the main loop while there are problems $(M)^i$ to be solved in the list Γ or else wherever there is still any *worker* in expansion work. If there is any problem in the list Γ, the *controller* choose one of them according to the *last-in-first-out* strategy and sends it to the first *worker* free. Having any expansion concluded, the *controller* receives the partial best upper bound U'_{BEST} and the list Γ' of recently generated children. It updates its own best upper bound U_{BEST} and its list Γ. Otherwise, the *controller* sends a terminate message to all *workers*. By themselves, the *workers* keep in a main loop receiving problems $(M)^i$, solving them, and sending back the results to the *controller* until they receive the terminate sign.

Because this implementation demands frequent synchronizations, it may cause a bottleneck in the *controller* and this could result in sub-utilization of the *worker* processes. However, it may be efficient if the granularity of the problem solved is coarse enough, *i.e.* if the problem expansion is computationally intense if compared to the communication costs as it appears to be the case here.

5 Concluding Remarks

A multi-level network optimization (MLNO) problem was presented and its importance in solving real world applications was discussed. The MLNO problems integrates location, topological network design and dimensioning aspects, being very convenient for use in the local access urban telecommunication network design problem. One possible mathematical programming formulation for the MLNO problem was presented and a branch-and-bound algorithm based on this formulation was used.

In order to increase the size of the manageable instances and to make larger practical networks tractable, future research might further explore reduction tests as well as parallel implementations. In fact, effective reduction tests were used to solve subproblems of the multi-level network optimization problem such as large uncapacitated location problems (Christofides and Beasley, 1982; Mateus and Carvalho, 1992) and Steiner problems in graphs (Maculan, Souza and Vejar, 1991). With the emerging emphasis on topological robustness and reliability, the study of enhanced models that incorporate connectivity constraints is a very promising area for investigations (Gendreau, Labbé and Laporte, 1995; Balakrishnan, Magnanti and Mirchandani, 1994).

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