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Flat Crush Strength Prediction

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Abstract

A statistical analysis on existing paper tube strength experimental data was performed in order to develop equations for flat crush prediction (FC) determinations. For each tube (a total of 108), information was available on paper grade, inside tube diameter (ID), spiral angle (β), P1A, P1B, FE1, FE2, P2 (all paper characteristics). It was of interest to predict flat crush based on these variables. The final equations for strength prediction are given by:

- mode 1 failure:

$$FC = 0.1143 * ID^{-0.5025} * \frac{P1A * P1B * FE1}{P1B(sin^{2.8879}\beta) + P1A(cos^{2.8879}\beta)}.$$

- mode 2 failure:

$$FC = 0.67 * ID^{-0.0831} * t^{-0.29} * P2 * FE2.$$

The "new" equations were compared with the equations currently in use. Some indication of violation of basic statistical assumptions wre found for the current equations. Consequently, important results such as: tests of significance and confidence intervals for the parameters; confidence intervals for the predicted flat crush strength, cannot be obtained. It was then recomended the use of the equations above for strength prediction.

KEY WORDS: *flat crush strength, nonlinear regression.*

1 Introduction/Objectives

The objective of this study is to perform statistical analysis on existing tube strength experimental data in order to develop equations for strength prediciton. In particular, it is of interest to predict flat crush strength (FC) based on several tube, design and paper properties, namely:

- inside diameter (ID) (in inches)
- thickness (t) (in inches)
- spiral angle (β)
- paper properties: P1A, P1B and P2 (in psi)
- tube properties: FE1, FE2

Two equations will be considered (known as "mode 1" and "mode 2" failures). They have been adapted from the ones used to predict strength on wood, and their formats are given below:

- mode 1 failure:

$$FC = k * \frac{P1A * P1B * FE1}{P1B(sin^n\beta) + P1A(cos^n\beta)} \tag{1}$$

- mode 2 failure:

$$FC = k * P2 * FE2 \tag{2}$$

It is also of interest to modify those equations in order to take into account a *size effect* given by $(\frac{k}{ID})^{1/n}$.

2 The Flat Crush Strength Data

The data available is on flat crush strength (FC) measurements for 108 tubes of different sizes: 2(two) inches inside diameter tubes (a total of 10), 3 in. ID tubes (74), 6 in. ID tubes (12) and 12 in. ID tubes (12).

For each tube, information is available on paper grade, inside diameter (ID), spiral angle (β), P1A, P1B, P2, FE1 and FE2. The values of FE1 and FE2 were obtained from a theoretical equation on ID, t and β. Information on the format of this equation was not provided. Although the *observed* ID values are not "exact" (i.e a 2 in. ID tube has actually an observed ID value of 1.998 in.), the *observed* values will be used since those were the ones used in the theoretical equations which produced the given values of FE1 and FE2.

3 Analysis/Results

In a first attempt to incorporate a size effect to the general equations (1) and (2), the researchers at Sonoco have developed the following ones:

- mode 1 failure

$$FC = 0.102 * ID^{-0.441} * \frac{P1A * P1B * FE1}{P1B(sin^{3.9}\beta) + P1A(cos^{3.9}\beta)}, \quad ID = 2, 3 \quad (3)$$

$$FC = 0.044 * ID^{-0.01} * \frac{P1A * P1B * FE1}{P1B(sin^{3.9}\beta) + P1A(cos^{3.9}\beta)}, \quad ID = 6, 12. \quad (4)$$

- mode 2 failure

$$FC = 1.04 * P2 * FE2 \quad for \quad ID = 2, 3 \quad (5)$$

$$FC = 4.48 * ID^{-0.886} * t^{0.424} * P2 * FE2 \quad for \quad ID = 6, 12. \quad (6)$$

The results obtained using the model developed by this statistical analysis ("new" model) will be compared with the ones obtained using the equations (3) to (6).

3.1 Mode 1 Failure

The format of the equation used for mode 1 failure, taking into account a size effect is given by:

$$FC = k * ID^{-1/n} * \frac{P1A * P1B * FE1}{P1B(sin^{m}\beta) + P1A(cos^{m}\beta)}.$$

The results of the model fitting are shown in Tables 1 and 2. The ANOVA table is in the Appendix.

parameter	estimate	asymptotic std.error	asymp.95%C.I	t ratio	p value
k	0.1142	0.0088	(0.0967,0.1318)	13.0	0.0000
n	1.9901	0.1310	(1.7304,2.2500)	15.2	0.0000
m	2.8879	0.7222	(1.4558,4.3199)	4.0	0.0000

Table 1: Parameter summary for the model fitted to the flat crush strength data -Mode 1 failure

The approximate t ratios are highly significant indicating that the parameters under consideration should not be deleted from the model. In addition, the parameter approximate correlation matrix (Table 2) does not exhibit any high correlation among the parameters. A high correlation among any two would indicate over-parametrization.

corr.	k	n	m
k	1.0000	-0.7708	0.8794
n	-0.7708	1.0000	-0.4040
m	0.8794	-0.4040	1.0000

Table 2: Asymptotic Correlation Matrix of the Parameters -Mod