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Forecast of Returns in Stock Markets by Jeffrey's Rule

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Abstract

The Bayes' formula is not always the appropriate way to link the prior and the posterior distributions. To calculate the posterior distribution by using Bayes' formula the posterior certainty about the conditioning event is required, what can be too strong in some situations. In this paper we consider the alternative procedure for the updating of probabilities presented by Jeffrey (1992,1965), which allows the addition of new information to the prior distribution under more circumstances than the Bayesian Conditioning. We present the predictivistic approach to Jeffrey's rule and establish the definition of conjugacy according to this rule (named Jeffrey-conjugacy). Results for Jeffrey-conjugacy in the exponential family are presented. As a sub-product these results provide predictivistic characterizations of some statistical models, similar to those presented by Arellano-Valle, Bolfarine and Iglesias (1994). By using the Jeffrey's rule and Jeffrey-conjugacy we also develop a forecasting procedure which is applied to the Chilean stock market data. We compare the Jeffrey's rule and the Bayesian Conditioning according their capability of incorporating unpredictable information to the forecast.

Keyword: Jeffey's rule, Bayesian Conditioning, Forecasting, Conjugacy, Predictivism.

1 Introduction

The usual Bayesian theory considers that the posterior distribution is obtained from the prior distribution by using the Bayesian Conditioning, that is, the posterior distribution is calculated by using the Bayes' formula. This procedure of probability updating demands that the probability attributed to the event E, after the receipt of new information, is 1, considering E as the conditioning event (Jeffrey, 1992 and Howson and Urback, 1993). However, the receipt of new information does not lead to that certainty in all situations. Under these circumstances, the use of the Bayesian conditioning is impossible. There are procedures of probability updating which are more flexible to the incorporation of new information. Some of them are discussed in Diaconis and Zabell (1982), Loschi, Iglesias and Wechsler (1998) and Jeffrey (1992).

The aim in this paper is to apply the Jeffrey's rule to forecast. The advantage of the proposed algorithm is to allow the addition of unpredictable information in more circumstances than with the Bayesian Conditioning.

In section 2, the Jeffrey's rule is presented and compared to the Bayesian Conditioning. In section 3, we use the generalized Jeffrey's rule to establish some results about the updating of probabilities within the predictivistic context. We observe that if the initial judgement about the prior distribution (exchangeability, for example) remains after the receipt of new information, the posterior distribution is obtained from the prior distribution by using Jeffrey's rule. In section 4, we establish the conjugacy according to the Jeffrey's rule (Jeffrey - conjugacy), providing particular results to the exponential family. In addition to examples of Jeffrey-conjugacy, the results related to the exponential family offer a predictivistic characterization (in the same sense presented by Arellano-Valle, Iglesias and Bolfarine (1994)) of some predictive distributions. Finally, in section 5, we apply the predictivistic Jeffrey's rule to the Chilean stock market data, comparing this rule and the Bayesian Conditioning in relation to their capability of incorporating unpredicted information to the previsions.

2 Definitions: Jeffrey's Rule

Jeffrey (1965) presents a new procedure for the updating of probabilities whose purpose is to provide a way to calculate posterior distributions, when the information received leads to a posterior probability less than 1 to the conditioning event. In such cases, it is impossible to use the Bayes' formula.

As defined onward, the Jeffrey's rule is less restrictive and so it can be used in more circumstances. Denote by P and P^* the prior and posterior distributions, respectively.

Definition 2.1 *(Jeffrey's Rule)*

Let $(\Omega, \mathcal{A})$ be a countable measurable space, P and P^* probability measures on it and $\mathcal{E} = \{E_i\}$ a partition of Ω. Assume that $\mathcal{E}$ is sufficient to $\{P, P^*\}$. We say that P^* is obtained from P by using the Jeffrey's rule if

$$P^*(A) = \sum_{i \geq 1} P(A|E_i)P^*(E_i), \quad A \in \mathcal{A}$$

where $P^*(E_i) \geq 0$, for every $i \geq 1$ and $\sum_{i=1} P^*(E_i) = 1$.

The link between Jeffrey's rule and Bayesian Conditioning can be easily noticed. If $P^*(E_{i_0}) = 1$ for some E_{i_0} in $\mathcal{E}$ and the J-condition can be considered valid, the posterior distribution obtained by using the Jeffrey's rule is

$$P^*(A) = P(A|E_{i_0}), \quad \forall\ A \in \mathcal{A},$$

what corresponds to an updating obtained by using the Bayesian Conditioning. (See Jeffrey (1992) for details).

To use any of these updating procedures, it is necessary to consider reasonable the condition of sufficiency of the partition $\mathcal{E}$ and construct new probabilities to events $E_i, i > 0$ in $\mathcal{E}$. But, if we consider the Jeffrey's rule as the updating procedure, the values declared for these posterior probabilities can be less than one - what can allow the incorporation of unanticipated information in a greater number of situations than using the Bayesian Conditioning. Notice that the condition of sufficiency of $\mathcal{E}$ mentioned in Definition 1.1, usually called J-condition, is a subjective evaluation, that is, there is no mathematical procedure to test the J-condition.

A classical example to illustrate the use of the Jeffrey's rule is given in Jeffrey (1965). In the situation described, the information received is obtained from sensory experiences, which rarely, or never, changes the observer's probability about some event of $\mathcal{E}$ to one.

Example 2.1 *You (Bayesian character) are in a dark room and do not remember the color of your own pajamas, building a sample space $\Omega = \{green,\ blue,\ white\}$ and having prior probability given by*

$$P(green) \ = \ 0.3, \ P(blue) \ = \ 0.4 \text{ and } P(white) \ = \ 0.3.$$

After a lightning strikes, your posterior probability for the event "the pajama is not green" is 0,8. Using the Jeffrey's rule, the posterior probabilities became

$$P^*(green) \ = \ 7/35, \ P^*(blue) \ = \ 16/35 \text{ and } P^*(white) = 12/35.$$

(Notice that in this situation the Bayesian Conditioning is not an appropriate procedure to obtain the posterior distribution. See more in Loschi, Iglesias and Wechsler (1998)).

Diaconis and Zabell (1982) present a generalization of this rule from the countable setting to the general space.

Definition 2.2 *(Generalized Jeffrey's Rule)*
Let $(\Omega, \mathcal{A}, P)$ be a probability space, P^* a new probability measure on $\mathcal{A}$ and $\mathcal{A}_0 \subseteq \mathcal{A}$ a sub-σ-algebra of $\mathcal{A}$. Let C be an $\mathcal{A}_0$-measurable set such that $P(C) = 0$ and $\bar{P}$ is absolutely continuous with respect to $\bar{P^*}$ on $\Omega - C$ where $\bar{P}$, $\bar{P^*}$ are the restrictions of P,P^* to $\mathcal{A}_0$. If $\mathcal{A}_0$ is sufficient to $\{P, P^*\}$, the Jeffrey's rule becomes:

$$P^*(A) = \int_{\Omega - C} P(A|\mathcal{A}_0)(w) dP^*(w) + P^*(A \cap C), \quad A \in \mathcal{A}$$

where $P(A|\mathcal{A}_0)$ is the conditional probability of A given $\mathcal{A}_0$. If P^* is absolutely continuous with respect to P, one can take $C = \Phi$.

Jeffrey (1992), Howson e Urback (1993) and Loschi (1998) present an extensive discussion about fundamental aspects of the Jeffrey's rule. Some mathematical properties of this updating procedure are also presented by Diaconis and Zabell (1982).

The predictivistic Jeffrey's rule is introduced in the next section, taking the more general approach to the Jeffrey's rule into account as well as considering $\Omega = \Omega_1, \times, \ldots, \times, \Omega_n$ where Ω_i, $i \geq 1$, is a measurable space.

3 Predictivistic Jeffrey's Rule

Diaconis and Freedman (1990) establish a general representation theorem for the exponential family extending previous results from Küchler and Lauritzen (1989). Informally, they establish that the predictive distribution of the sequence $X_1, \ldots, X_n$ is a mixture of distributions P_θ in the exponential family, if the conditional distribution of $X_1, \ldots, X_n$, given $X_1 + \ldots + X_n$ is the same conditional distribution of $Y_1, \ldots, Y_n$, given $Y_1 + \ldots + Y_n$, where Y_i is distributed according to P_θ.

We start by considering the less general situation described by Diaconis and Freedman (1990) to understand the Jeffrey's rule within the predictivistic setting.

Let h be a non-negative, finite, locally integrable Borel function on $\mathcal{R}^d$, where $\mathcal{R} = (-\infty, \infty)$ and d is a fixed, positive integer. Let

$$D(\theta) = \int_{\mathcal{R}^d} \exp\{x \cdot \theta\} h(x) dx \quad \text{for} \quad \theta \in \mathcal{R}^d \tag{1}$$

where $x \cdot \theta$ denotes the usual inner product of x by θ on $\mathcal{R}^d$. Consider the following set

$$\Theta = \{\theta \in \mathcal{R}^d : D(\theta) < \infty\}, \tag{2}$$

which is supposed to be a non-empty, convex subset of $\mathcal{R}^d$.

Consider the probability measure P as a member of the exponential family and assume that the density function for P is given by

$$dP_\theta(x) = \frac{\exp\{x \cdot \theta\} h(x)}{D(\theta)} dx, \quad \forall \theta \in \Theta \subseteq \mathcal{R}^d. \tag{3}$$

Let $X_1, X_2, \ldots$ be a sequence of random quantities assuming values in $\mathcal{R}^d$ and $S_n = X_1 + \ldots + X_n$. Define the probability measure P_θ^∞ on $(\mathcal{R}^d)^\infty$ so that it makes the random quantities X_i's independent and having the common distribution P_θ.

Consider $C_n = \{s \in \mathcal{R}^d : 0 < h^{(n)}(s) < \infty\}$ where $h^{(j)}$ denotes the j-fold convolution of h with itself. Let $t = x_1 + \ldots + x_k$. Suppose $k < n$ and choose $s \in C_n$. Thus, the conditional distribution Q_{nsk} for $X_1, \ldots, X_k$ given $S_n = s$ is the absolutely continuous distribution on $\mathcal{R}^d$ with density

$$\frac{h(x_1) \ldots h(x_k) h^{(n-k)}(s - t)}{h^{(n)}(s)}.$$

If $k = n$, Q_{nsk} is a singular distribution.

Notice that Q_{nsk} is a regular conditional distribution for $X_1, \ldots, X_k$ given $S_n = s$, relative to P_θ^∞ for every $\theta \in \Theta$; therefore S_n is a sufficient statistics for θ.

Denote by $\mathcal{M}_Q$ the family of all absolutely continuous (with respect to the Lebesgue measure) probability measures P defined on $(\mathcal{R}^d)^\infty$ such that for every $n \geq 1$,

(i) $P(S_n \in C_n) = 1$

(ii) Q_{nsn} is a regular conditional P-law for $X_1, \dots, X_n$ given $S_n = s$.

Theorem 3.1 *(Diaconis e Freedman(1990))*

Suppose that h is a non-negative, finite, locally integrable Borel function on $\mathcal{R}^d$ and assume the conditions (1) and (2). Then $P \in \mathcal{M}_Q$ if, and only if,

$$P = \int_\Theta P_\theta^\infty dQ(\theta). \tag{4}$$

The probability measure Q is uniquely defined on the Borel subsets of the convex set $\Theta \subset \mathcal{R}^d$.

Consequently, every sub-sequence $X_{(n)} = (X_1, \dots, X_n)$, $n \geq 1$ is represented by

$$P(X_1 \leq x_1, \dots, X_n \leq x_n) = \int_\Theta P_\theta^n(X_1 \leq x_1, \dots, X_n \leq x_n) dQ(\theta), \tag{5}$$

where P_θ^n denotes the P-law which makes $X_1, \dots, X_n$ independent and identically distributed, having the distribution P_θ.

Assuming that P_θ is absolutely continuous with respect to the σ-finite, positive measure λ, the above result is expressed in terms of densities, that is, the joint density of $X_1, \dots, X_n$ is

$$p(x_1, \dots, x_n) = \int_\Theta p_\theta(x_1, \dots, x_n) dQ(\theta), \quad \forall\, n \geq 1, \tag{6}$$

where p and p_θ are density functions with respect to λ. The next example illustrates the use of Theorem 3.1.

Example 3.1 *Let $X_1, X_2, \dots$ be an infinite sequence of positive, random variables such that*

$$P(X_{(k)} \in A) = P(X_{(k)} \in A + \mathbf{x}), \; \forall\, k \geq 1 \tag{7}$$

where A is a Borel set of $\mathcal{R}_+^k$ and $\mathbf{x} \in \mathcal{R}^k$ is such that $\sum_{i=1}^k x_i = 0$ and $A + \mathbf{x} \in \mathcal{R}_+^k$. Consider $S_n = X_1 + \ldots + X_n$. Then

$$X_1, \dots, X_k | S_n = s \sim \mathcal{U}\left\{ (x_1, \dots, x_k) \in \mathcal{R}_+^k : \sum_{i=1}^k x_i = s \right\} = Q_{nsk}, \quad s > 0. \tag{8}$$

Observe that Q_{nsk} is also the law of $Y_1, \dots, Y_k$ given $Y_1 + \ldots + Y_n = s$, if the random variables Y_i's are independent and identically distributed, having the exponential distribution $\mathcal{E}(\theta)$, $\theta > 0$, whose density function is

$$dP_\theta(x) = \exp\{-\theta x + log(\theta)\} 1_{\{x>0\}} dx. \tag{9}$$

Notice that the density in (9) is a member of the exponential family with $D(\theta) = 1/\theta$, $\Theta = (0, \infty)$ and $h(x_i) = 1_{\{x_i > 0\}}$ for every i. Consequently, the density induced by Q_{nsk} is

$$f_{Q_{nsk}} = \frac{h^{(n-k)}(s-t)}{h^{(n)}(s)} = \frac{(n-1)!}{(n-k-1)!} \frac{(s - \sum_{i=1}^{k} x_i)^{n-k-1}}{s^{n-1}}$$

Besides this, we can observe that S_n is a sufficient statistics for θ and $C_n = (0, \infty)$, what makes $P(S_n \in C_n) = 1$. As a consequence of Theorem 3.1, there is a probability measure Q on the Borel set of $\mathcal{R}_+$, such that

$$\begin{aligned} p(x_1, \ldots, x_n) &= \int_0^\infty \theta^n \exp\left\{ -\theta \sum_{i=1}^{n} x_i \right\} dQ(\theta) \\ &= \int_0^\infty \Pi_{i=1}^{n} \theta \exp\{-\theta x_i\} \, dQ(\theta), \ \forall \ n \geq 1. \end{aligned} \tag{10}$$

The Jeffrey's rule within the setting considered in Theorem 3.1 is obtained as follows:

Proposition 3.1 *Suppose that h is a non-negative, finite, locally integrable Borel function on $\mathcal{R}^d$ and assume the conditions (1) and (2). Let P and P^* be probability measures in $\mathcal{M}_Q$. Then P^* is obtained from P by means of the Jeffrey's rule with respect to the sub-σ-algebra $\mathcal{A}_0$ generated by $\lim_{n\to\infty} S_n/n$.*

Proof: From Theorem 3.1, it follows that for every probability measure $P' \in \mathcal{M}_Q$ there is a unique probability measure Q' defined on the Borel sets of $\Theta \subseteq \mathcal{R}^d$, such that

$$\begin{aligned} P'(X_1 \leq x_1, \ldots, X_n \leq x_n) &= \int_\Theta P_\theta^n(X_1 \leq x_1, \ldots, X_n \leq x_n) dQ'(\theta), \ \forall \ n \geq 1 \qquad (11) \\ &= \int_\Theta \Pi_{i=1}^{n} P_\theta(X_i \leq x_i) dQ'(\theta), \end{aligned}$$

where, for each $\theta \in \Theta$, $\Pi_{i=1}^{n} P_\theta(X_i \leq x_i) = P'(X_1 \leq x_1, \ldots, X_n \leq x_n | \mathcal{A}_0)(x_1, x_2, \ldots)$ for every $P' \in \mathcal{M}_Q$ and $\mathcal{A}_0 = \sigma(\theta) = \sigma(\lim_{n\to\infty} S_n/n)$.

Notice that $\mathcal{A}_0$ is a sufficient sub-σ-algebra for the family $\mathcal{M}_Q$ and, in particular, sufficient for $\{P, P^*\} \subseteq \mathcal{M}_Q$. Then,

$$\begin{aligned} P^*(X_1 \leq x_1, \ldots, X_n \leq x_n) &= \int_{(\mathcal{R}^d)^\infty} P(X_1 \leq x_1, \ldots, X_n \leq x_n | \mathcal{A}_0)(x_1, x_2, \ldots) dP^*(x_1, x_2, \ldots) \\ &= \int_\Theta \Pi_{i=1}^{n} P_\theta(X_i \leq x_i) dQ^*(\theta). \end{aligned} \tag{12}$$

It follows from expression (12) that P^* is absolutely continuous with respect to P, concluding the proof.

□

The expression (12) provides a predictivistic approach to the Jeffrey's rule for probability laws that can be represented as a mixture of distributions in the exponential family. Notice that if the prior judgement on the observable quantities, which for Proposition 3.1 is $P \in \mathcal{M}_Q$, remains after the receipt of new information (sometimes unanticipated information), the posterior distribution can be obtained by the Jeffrey's rule. In these cases, it is only necessary to establish the posterior distribution Q^* for θ.

For example, if in Example 3.1 the receipt of a new piece of information produces a change on the prior distribution for θ but the judgement about $X_{(k)}$ presented in (7) is kept, then the posterior distribution for every finite sub-sequence $X_1, \ldots, X_n$ is obtained by the Jeffrey's rule and has the following density function:

$$p^*(x_1, \ldots, x_n) = \int_0^\infty \Pi_{i=1}^n \theta \exp\{-\theta x_i\}\, dQ^*(\theta),\ n \geq 1,$$

where Q^* represents the posterior distribution for θ. Observe that the prior evaluation of the $X_1, \ldots, X_n$, $n > 0$, is only partially changed through a complete reassessment. The result presented in Proposition 3.1 can be extended to more general cases if we consider theorem 4.4 from Diaconis and Freedman(1990)(see section 4).

Similar results are obtained if some prior judgement of invariance (i.e, exchangeability, spherical symmetry, and so on) remains unchangeable in the posterior evaluation. In these cases, the existence of de Finetti-style representation theorems warrants the existence of a sufficient sub-σ-algebra for $\{P, P^*\}$.

Example 3.2 *Let $X_1, X_2, \ldots$ be an infinite sequence of 0-1 random variables, with exchangeable P-law. Consider the prior distribution Q for $\theta = \lim_{n\to\infty} \sum_{i=1}^n X_i/n$. From de Finetti's theorem (1937) it follows that*

$$P(X_1 = x_1, \ldots, X_n = x_n) = \int_\Theta \Pi_{i=1}^n \theta^{x_i}(1-\theta)^{1-x_i}\, dQ(\theta).$$

If the posterior judgment about the sequence $X_1, X_2, \ldots$ reveals that exchangeability remains and that Q is updated to a new distribution Q^*, the posterior distribution for every sub-sequence $X_1, \ldots, X_n$ can be obtained by the Jeffrey's rule as follows:

$$P^*(X_1 = x_1, \ldots, X_n = x_n) = \int_\Theta \Pi_{i=1}^n \theta^{x_i}(1-\theta)^{1-x_i}\, dQ^*(\theta). \tag{13}$$

4 Jeffrey - Conjugacy

According to the usual Bayesian point of view, if the prior and the posterior distribution are members of the same family of probability measures, we say that these distributions are conjugate with respect to the likelihood function which updated the prior distribution. Within this setting, the likelihood function contains all the information needed to update the prior distribution.

However, when the posterior distribution is obtained by using the Jeffrey's rule, part of the information is included on the prior distribution through a complete reassessment, that is, the likelihood function contains just a piece of information useful to the updating of the prior distribution.

In this section some conditions are established to obtain the conjugacy in case the posterior distribution is obtained by using the Jeffrey's rule.

Let $\mathcal{P}$ be a family of probability measures on $(\Omega, \mathcal{A})$ and $\mathcal{A}_0$ a sub-σ- algebra of $\mathcal{A}$. Consider $P \in \mathcal{P}$ and P^* a probability measure on $(\Omega, \mathcal{A})$. Suppose that P^* is absolutely continuous with respect to P and is obtained from P by using the Jeffrey's rule, with respect to $\mathcal{A}_0$. If $P^* \in \mathcal{P}$, we say that $\mathcal{P}$ is a **Jeffrey-conjugate family**, with respect to the sub-σ-algebra $\mathcal{A}_0$.

Notice that the family $\mathcal{M}_Q$ of absolutely continuous probability measures on $(\mathcal{R}^d)^\infty$, defined in Theorem 3.1, is Jeffrey -conjugated with respect to the sub-σ-algebra $\mathcal{A}_0$ generated by $\lim_{n\to\infty} S_n/n$. Similarly, we can prove that every probability measure dominated by a exchangeable measure, and obtained from this measure by using the Jeffrey's rule, is exchangeable.

Despite this, Jeffrey-conjugacy is too general and sometimes leads to undesirable situations. Notice that the family of all probability measures on $(\Omega, \mathcal{A})$ is Jeffrey-conjugated with respect to the σ-algebra $\mathcal{A}$ of the parts of Ω. In this case, the posterior distribution P^* consists in the complete and arbitrary reassessment of the opinion.

Adding some conditions to the family of the mixing measures Q, we can obtain a more interesting definition to conjugacy according to the Jeffrey's rule - the **natural Jeffrey-conjugacy**.

Definition 4.1 *(Natural Jeffrey-Conjugated Family)*
Let $\mathcal{P}$ be a family of probability measures on the measurable space $(\Omega, \mathcal{A})$ and $\mathcal{A}_0 \subseteq \mathcal{A}$ the sub-σ- algebra generated by the random quantity $\theta = \theta(w)$, $w \in \Omega$. Assume that $\theta \in \Theta$, where Θ is a finite parametric space. We say that the family $\mathcal{P}$ is a Natural Jeffrey-conjugated one with respect to $\mathcal{A}_0$ if

(i) *$\mathcal{P}$ is Jeffrey-conjugated with respect to $\mathcal{A}_0$, that is,*

$$P^*(\cdot) = \int_\Theta P(\cdot|\mathcal{A}_0)(w)dQ^*(\theta) \quad \in \quad \mathcal{P}, \tag{14}$$

where $Q^* \in \mathcal{Q}$ is the restriction of P^* to the Borel sets of Θ and $\mathcal{Q}$ is the family of probability measures defined on Θ;

(ii) *the family $\mathcal{Q}$ is natural-conjugated (in the usual Bayesian sense) with respect to $P(.|\mathcal{A}_0)$.*

Notice that if the natural Jeffrey-conjugacy is considered, the number of elements of $\mathcal{P}$ is reduced without making the choice of the prior distribution imperative.

Example 4.1: *Notice from Example 3.2 that the family of exchangeable probability measures on $(\{0,1\}^\infty, \mathcal{B}^\infty)$ is Jeffrey-conjugated with respect to the sub-σ-algebra $\mathcal{A}_0$ generated by θ. In this case P^* is given by (13). Consequently, if Q and Q^* are Beta distributions, the family $\mathcal{P}$ is natural Jeffrey-conjugated with respect to $\mathcal{A}_0$ and*

$$P^*(X_1 = x_1, \ldots, X_n = x_n) = \frac{\mathrm{Beta}(\alpha^* + \sum_{i=1}^n x_i, n + \beta^* - \sum_{i=1}^n x_i)}{\mathrm{Beta}(\alpha^*, \beta^*)}, \tag{15}$$

where $\mathrm{Beta}(a, b)$ *is the Beta function with parameters a and b.*

It can be proved that the family $\mathcal{M}_Q$ defined in Theorem 3.1 is a natural Jeffrey-conjugated family with respect to the sub-σ-algebra $\mathcal{A}_0$ generated by $\lim_{n\to\infty} S_n/n$ if, and only if, the family of mixing measures considered in that theorem is natural-conjugate with respect to the likelihood function P_θ^∞ induced by $\mathcal{A}_0$. It means that if the prior and posterior joint distributions of a sequence of random quantities are represented as mixtures of distributions in the exponential family with parameter θ, the natural Jeffrey-conjugacy is obtained when the posterior distribution for θ is a result of a free choice on the family to which the prior distribution for θ belongs .

Assuming the conditions established in Theorem 2.1 from Diaconis and Freedman(1990), and supposing that θ has a proper prior distribution Q, which is not concentrated at a single point, the natural Jeffrey-conjugacy can be obtained within a predictivistic setting as presented in the following result:

Proposition 4.1 *Let $X_1, X_2, \ldots$ be an infinite sequence of absolutely continuous random quantities defined on $\mathcal{R}^d$ with law $P \in \mathcal{M}_Q$. If*

$$E(X_2|X_1) = aX_1 + b, \tag{16}$$

for any constant a and any constant vector b with dimension d, it follows that $a \neq 0$ and

$$p(x_1,\ldots,x_n) = \Pi_{i=1}^n h(x_i) \frac{k\left(n + \frac{1-a}{a}; \frac{b}{a} + \sum_{i=1}^n x_i\right)}{k\left(\frac{1-a}{a}; \frac{b}{a}\right)}, \ \forall n \geq 1, \tag{17}$$

where $k(\tau_0, \tau_1) = \int_\Theta D(\theta)^{-\tau_0} \exp\{\theta \cdot \tau_1\} d\theta$ and p is the density function induced by P.

Proof: As $P \in \mathcal{M}_Q$, it follows that

$$p(x_1,\ldots,x_n) = \int_\Theta \Pi_{i=1}^n h(x_i)[D(\theta)]^{-n} \exp\left\{\sum_{i=1}^n x_i \cdot \theta\right\} dQ(\theta), \quad \forall n \geq 1. \tag{18}$$

Let X_1 be a sample of size one from the distribution given in (3). Consider $d\xi(x) = h(x)dx$ and $L(\theta) = \log D(\theta)$. Then, from the properties of the exponential family we obtain:

$$E(X_1|\theta) = \frac{d}{d\theta} L(\theta) = L'(\theta). \tag{19}$$

Using the hypotheses and the properties of conditional expectation, and also noticing that X_1 and X_2, conditionally in θ, are independent and have a common distribution P_θ, it follows that

$$\begin{aligned} E\{L'(\theta)|X_1\} &= E\{E\{X_1|\theta\}|X_1\} \\ &= E\{E\{X_2|\theta\}|X_1\} \\ &= E\{E\{X_2|\theta, X_1\}|X_1\} \\ &= E\{X_2|X_1\} = aX_1 + b. \end{aligned}$$

Therefore, as it is supposed that θ has a proper distribution which is not concentrated at a single point, from Diaconis and Ylvisaker's theorem (1979) it can be concluded that Q is absolutely continuous with respect to the Lebesgue measure and is natural-conjugated with respect to P_θ^∞. Besides this, the density function induced by Q is

$$dQ(\theta) = c \exp\left\{ \frac{b}{a} \cdot \theta - \frac{1-a}{a} \log D(\theta) \right\} d\theta, \tag{20}$$

where $c = \frac{1}{k(\frac{1-a}{a}; \frac{b}{a})}$ and $k(\tau_0, \tau_1) = \int_\Theta D(\theta)^{-\tau_0} \exp\{\theta \cdot \tau_1\} d\theta$.

Consequently, the predictive distribution to $X_1, \ldots, X_n$ given in (17) can be obtained from the expressions (18) and (20).

□

Let $\mathcal{L}$ be the family of absolutely continuous probability measures P on $(\mathcal{R}^d)^\infty$ such that

$$E_P(X_2|X_1) = aX_1 + b,$$

for some $a \in \mathcal{R} - \{0\}$ and $b \in \mathcal{R}^d$. Note that if $P^* \in \mathcal{L}$ is absolutely continuous with respect to $P \in \mathcal{M}_Q \cap \mathcal{L}$, and is obtained from P by using the Jeffrey's rule, then $\mathcal{M}_Q$ is a natural Jeffrey-conjugated family with respect to $\mathcal{A}_0 = \sigma(\lim_{n\to\infty} S_n/n)$.

Example 4.2: *(Example 3.1 - continuation) Consider an infinite sequence of random variables $X_1, X_2, \ldots$, with P-law satisfying the condition (7) and pertaining to the family $\mathcal{L}$ composed by all absolutely continuous probability measures P on $\mathcal{R}$ and such that $E_P(X_2|X_1) = aX_1 + b$ for any $a, b \in \mathcal{R}$, $a \neq 0$, and $P(X_1 = 0) = 0$. Then, from (10) it follows that the predictive distribution for $X_1, \ldots X_n$ is*

$$p(x_1, \ldots, x_n) = \int_0^\infty \Pi_{i=1}^n \theta \exp\{-\theta x_i\} dQ(\theta), \ \forall \ n \geq 1. \tag{21}$$

Consequently, the density function induced by the prior distribution Q is a proper distribution which is not concentrated at a single point and has the following density:

$$dQ(\theta) = \frac{1}{k\left(\frac{1-a}{a}; \frac{b}{a}\right)} \theta^{\frac{1-a}{a}} \exp\left\{ \frac{b}{a} \cdot \theta \right\}, \ \theta > 0,$$

where $k\left(\frac{1-a}{a}; \frac{b}{a}\right) = \frac{\Gamma(1/a)}{(b/a)^{1/a}}$.

If the receipt of a piece of unanticipated information leads to the changing of the prior opinion just on a and b, the posterior distribution $P^* \in \mathcal{L}$ is obtained from P by the Jeffrey's rule and, therefore, $P^* \in \mathcal{M}_Q$. From Proposition 4.1, it follows that

$$dQ^*(\theta) = \frac{1}{k\left(\frac{1-a^*}{a^*}; \frac{b^*}{a^*}\right)} \theta^{\frac{1-a^*}{a^*}} \exp\left\{ \frac{b^*}{a^*} \cdot \theta \right\}, \ \theta > 0.$$

where $k\left(\frac{1-a^*}{a^*}; \frac{b^*}{a^*}\right) = \frac{\Gamma(1/a^*)}{(b^*/a^*)^{1/a^*}}$.

Hence, the family $\mathcal{M}_Q$, considered in Example 3.1, is natural Jeffrey-conjugated with respect to $\mathcal{A}_0 = \sigma(\lim_{n\to\infty} \sum_{i=1}^n X_i/n)$.

Example 4.2 shows a situation where natural Jeffrey-conjugacy is basically a consequence of judgements about observable quantities. Some similar examples can be obtained by using results established in Arellano-Valle, Iglesias and Bolfarine(1994) and Loschi, Iglesias and Arellano-Valle(1999).

The result presented in Proposition 4.1 provides a characterization of the predictive distributions when the sample and parametric spaces have the same dimension. If, for example, for an infinite sequence of random variables $X_1, X_2, \ldots$, the conditional distribution of $X_1, \ldots, X_n$, given $\sum_{i=1}^n X_i$ and $\sum_{i=1}^n X_i^2$, is the same conditional distribution as that for a sequence of random variables having a normal distribution with mean μ and variance σ^2, neither Theorem 3.1, nor the results concerning the Jeffrey's rule, obtained from it, apply. For these cases, the extension of theorem 2.1 presented by Diaconis and Freedman(1990) is considered.

Let $(\Omega, \mathcal{A})$ be a Polish space and let ρ be a σ-finite reference measure on $(\Omega, \mathcal{A})$. Define the exponential family $\{P_\theta : \theta \in \Theta\}$ on $(\Omega, \mathcal{A})$ with a Borel measurable sufficient statistics $\Phi : \Omega \to \mathcal{R}^d$ as follows. Let

$$D(\theta) = \int_\Omega \exp\{\theta \cdot \Phi(w)\} d\rho(w).$$

Consider $\Theta = \{\theta : \theta \in \mathcal{R}^d, D(\theta) < \infty\}$ a convex subset of $\mathcal{R}^d$. For $\theta \in \Theta$, let

$$dP_\theta(w) = \frac{1}{D(\theta)} \exp\{\theta \cdot \Phi(w)\} d\rho(w). \tag{22}$$

Suppose that:

(A) for some positive integer ν, the ρ^ν-law of $\Phi(w_1) + \ldots + \Phi(w_\nu)$ is absolutely continuous in $\mathcal{R}^d$, with locally integrable density h, and

(B) Θ is non-empty.

Notice that $\theta \in \Theta$ iff $\int_{\mathcal{R}^\nu} \exp(\theta \cdot x) h(x) dx < \infty$.

Let X_i be the coordinate function on Ω^∞. Let P_θ^∞ on $\mathcal{A}^\infty$ be the probability measure that makes $X_1, X_2, \ldots$ independent, with a common distribution P_θ. Let $S_n = \Phi(X_1) + \ldots + \Phi(X_n)$. The P_θ^∞-law of S_ν is absolutely continuous in $\mathcal{R}^d$, with density $\frac{1}{D(\theta)^\nu} \exp\{\theta \cdot x\} h(x)$.

Let Q_{ns} be a regular conditional distribution for $X_1, \ldots, X_n$ given $S_n = s$, defined for $s \in C_n$, where Q_{ns} and C_n are constructed so that for every n:

(i) $P_\theta(S_n \in C_n) \ = \ 1$ for every θ, and Q_{ns} works for every P_θ.

(ii) $Q_{ns}(S_k \in C_k) = 1$ for every $s \in C_n$ and $k = 1, \ldots, n$.

(iii) Q_{k,S_k} is a regular conditional distribution for $X_1, \ldots, X_k$ given S_k and $X_{k+1}, \ldots, X_n$, computed relative to Q_{ns}, for every $s \in C_n$ and every $k = 1, \ldots, n-1$.

Let P be a probability measure on $(\Omega^\infty, \mathcal{A}^\infty)$. Say $P \in \mathcal{M}_Q$ iff for every $n = 1, 2, \ldots$, $P(S_n \in C_n) = 1$ and Q_{ns} is a regular conditional distribution for $X_1, \ldots, X_n$ given $S_n = s$, relative to P.

Considering this setting, Diaconis and Freedman(1990) prove the result that follows, which represents, for example, random variables as a mixture of distributions in the exponential family with two or more parameters.

Theorem 4.1 *(Diaconis and Freedman(1990))- Suppose the conditions (A) and (B) given. Then* $P \in \mathcal{M}_Q$ *iff* $P = \int_\Theta P_\theta^\infty d\mu(\theta)$ *for a unique probability measure* μ *on the Borel subsets of the convex set* $\Theta \subset \mathcal{R}^d$.

Assume the conditions in Theorem 4.1 and suppose that θ has a proper distribution which is not concentrated at a single point.

Proposition 4.2 *Let* $X_1, X_2, \ldots$ *be an infinite sequence of random quantities defined on* Ω*, whose* P*-law belongs to the family* $\mathcal{M}_Q$*. Suppose that*

$$E(\Phi(X_2)|X_1) = a\Phi(X_1) + b \tag{23}$$

for some $a \in \mathcal{R}$ *and* $b \in \mathcal{R}^d$*. Then, for every* $n \geq 1$ *it follows that* $a \neq 0$ *and*

$$p(x_1, \ldots, x_n) = \Pi_{i=1}^n h(x_i) \frac{k(n + \frac{1-a}{a}; b + \sum_{i=1}^n \Phi(x_i))}{k(\frac{1-a}{a}; \frac{b}{a})}, \tag{24}$$

where p *is the density function with respect to the Lebesgue measure induced by* P *and*

$$K(\tau_0, \tau_1) = \int_\Theta [D(\theta)]^{-\tau_0} \exp\{\theta \cdot \tau_1\} d(\theta).$$

Proof: From Theorem 4.1, there is a probability measure μ on the Borel sets of Θ such that

$$P(X_1 \leq x_1, \ldots, X_n \leq x_n) = \int_\Theta P_\theta^\infty(X_1 \leq x_1, \ldots, X_n \leq x_n) d\mu(\theta),$$

where P_θ^∞ is the P-law which makes $X_1, X_2, \ldots$ be independent and have the common distribution P_θ, whose density function is given in (22).

The Borel-measurable function Φ is a sufficient statistics for $\{P_\theta : \theta \in \Theta\}$. Thus, under P_θ^∞, $\Phi(X_1), \Phi(X_2), \ldots$ is a sequence of independent and identically distributed random quantities with the common density function

$$dP_{\theta\Phi(X_i)}(t) = \frac{1}{D(\theta)} \exp\{\theta \cdot t\} h(t) dt, \;\; t \in \mathcal{R}^d, i \geq 1. \tag{25}$$

As $dP_{\theta\Phi(X_i)}$ belongs to the exponential family, it follows that $E(\Phi(X_1)|\theta) = \frac{d}{d\theta} L(\theta) = L'(\theta)$, where $L(\theta) = \log D(\theta)$. Consequently, using properties of conditional expectation, it follows that

$$\begin{aligned} E\{L'(\theta)|\Phi(X_1)\} &= E\{E\{\Phi(X_1)|\theta\}|\Phi(X_1)\} \\ &= E\{E\{\Phi(X_2)|\theta\}|X_1\} \\ &= E\{E\{\Phi(X_2)|\theta, X_1\}|X_1\} \\ &= E\{\Phi(X_2)|X_1\} = a\Phi(X_1) + b. \end{aligned}$$

Therefore, from Diaconis and Ylvisaker's theorem (1979), it follows that μ is absolutely continuous with density function

$$d\mu(\theta) = c \exp\left\{\frac{b}{a} \cdot \theta - \frac{1-a}{a} \log D(\theta)\right\} d(\theta), \tag{26}$$

where $c^{-1} = k\left(\frac{1-a}{a}; \frac{b}{a}\right)$.

¿From (25) and (26) it follows that the predictive distribution for $X_1, \ldots, X_n$ has the density function given in (24).

$\square$

Example 4.3: *Suppose that $X_1, X_2, \ldots$ are random variables such that the distribution of $X_1, \ldots, X_n$ given $(\sum_{i=1}^n X_i = m, \sum_{i=1}^n X_i^2 = s)$ is the same as that for the random variables which are distributed according to the normal distribution $\mathcal{N}(\mu, \sigma^2)$. From Theorem 4.1 it follows that for every $n \geq 1$*

$$p(x_1, \ldots, x_n) = \int_\Theta \Pi_{i=1}^n \frac{1}{\sigma(2\pi)^{1/2}} \exp\left\{-1/2 \frac{(x_i - \mu)^2}{2\sigma^2}\right\} dQ(\mu, \sigma^2)$$

where $\Theta = \{(\mu, \sigma^2) : \mu \in \mathcal{R}, \sigma > 0\}$. Suppose that $P(X_1 = X_2) = 0$ and

$$E_P((X_3, X_3^2)|(X_1, X_2)) = a(X_1 + X_2; X_1^2 + X_2^2) + (b_1, b_2). \tag{27}$$

Hence, considering Proposition 4.2, $a \in (0, 1/2)$, $b_1 \in \mathcal{R}$, $b_2 > \frac{b_1^2}{1-2a}$ and the prior distribution to (μ, σ^2) is the following Normal-Inverted-Gamma distribution:

$$\mu|\sigma^2 \sim \mathcal{N}\left(\frac{b_1}{1-2a}, \frac{a\sigma^2}{1-2a}\right) \quad \text{and} \quad \sigma^2 \sim \mathcal{GI}\left(\frac{1+a}{2a}, \frac{1}{2a}\left(b_2 - \frac{b_1^2}{1-2a}\right)\right). \tag{28}$$

Consequently, the predictive distribution to the subsequence $X_1, \ldots, X_n$ is the n-variate Student-t distribution

$$(X_1, \ldots, X_n) \sim t_n\left(\frac{b_1 \mathbf{1}_n}{1-2a}, \mathbf{I}_n + \frac{a \mathbf{1}_n \mathbf{1}_n^t}{1-2a}; \frac{1}{a}\left(b_2 - \frac{b_1^2}{1-2a}\right), \frac{1+a}{a}\right), \quad n \geq 3. \tag{29}$$

Remark: *Smith(1981) proves that if an infinite sequence of random variables has a law within the family of probability distributions which are invariant under the group of orthogonal transformations that preserve vectors of ones (that is, it is $O(\mathbf{1})$-invariant), all subsequences can be represented as the mixture of $\mathcal{N}(\mu, \sigma^2)$ distributions performed by the appropriate prior distribution (mixing measure) to (μ, σ^2). Consequently, if the condition given in (27) is also considered, the same predictive and prior distributions as in Example 4.3 are obtained. (The complete proof of this fact can be found in Loschi, Iglesias and Arellano-Valle(1998).)*

Results related to Jeffrey-conjugacy when Ω is a Polish space are similar to those obtained for a more simple space. In the next section, the Jeffrey's rule is applied to forecast returns in the Chilean stock market . The procedure presented permits the incorporation of unanticipated information in more circumstances than does the Bayesian conditioning.

5 Applications

The aim in this section is to show that the forecasts obtained by using the Jeffrey's rule are better than those obtained by using Bayesian conditioning. One reason for this is the flexibility of the Jeffrey's rule in the incorporation of unpredictable information on the prior distribution.

Denote by $X_{(k)}$ the random vector $(X_1, \ldots, X_k)$. In the examples to be presented the predictivistic approach to the Jeffrey's rule is considered and the natural Jeffrey-conjugacy assumed. In fact, the Student-t distribution given in the expression (29) is adopted to model the behavior of random variables which are in the same block. These assumptions seem to be reasonable to the Chilean stock market as it is explained in example 5.2.

Consequently, if no unpredictable information is received, forecasts are obtained by considering the following conditional distribution:

$$X_{(n-k)}|X_{(k)} = \mathbf{x}_k \sim t_{n-k}(\mu_{n-k}(\mathbf{x}_k); \Sigma_{n-k\cdot k}; \lambda_{q(\mathbf{x}_k)}, \nu_{n-k}) \tag{30}$$

where

$\mu_{n-k}(\mathbf{x}_k) = \{(a \sum_{i=1}^{k} x_i + b_1)/(1 + a(k-2))\}\mathbf{1}_{n-k\times 1};$

$\Sigma_{n-k\cdot k} = I_{n-k} + \frac{a}{1+a(k-2)}\mathbf{1}_{n-k\times n-k};$

$q(x_k) = \sum_{i=1}^{k} x_i^2 + \frac{b_1^2 k}{(1-2a)(1+a(k-2))} - \frac{a}{1+a(k-2)}\,(\sum_{i=1}^{k} x_i)^2 - \frac{2b_1}{1+a(k-2)}\,\sum_{i=1}^{k} x_i;$

$\lambda q(x_k) = \frac{b_2}{a} - \frac{b_1^2}{a(1+a(k-2))} + \sum_{i=1}^{k} x_i^2 - \frac{a}{1+a(k-2)}(\sum_{i=1}^{k} x_i)^2 - \frac{2b_1}{1+a(k-2)}\sum_{i=1}^{k} x_i;$ and

$\nu_{n-k} = \frac{1+a(k+1)}{a}.$

The one-step forecasts made at the k-th instant and the variances associated to these forecasts are given respectively by

$$\begin{aligned} E(X_{k+1}|X_{(k)}) &= \frac{a\sum_{i=1}^{k} X_i + b_1}{1+a(k-2)} \quad \text{and} \\ \mathrm{Var}(X_{k+1}|X_{(k)}) &= \frac{a}{1+a(k-2)}\left\{\frac{b_2}{a} - \frac{b_1^2}{a(1+a(k-2))} + \sum_{i=1}^{k} X_i^2 - \frac{a}{1+a(k-2)}(\sum_{i=1}^{k} X_i)^2 - \frac{2b_1\sum_{i=1}^{k} X_i}{1+a(k-2)}\right\}. \end{aligned} \tag{31}$$

Suppose that some unanticipated information is made known at the k-th instant and assume the natural Jeffrey-conjugacy - that is, the posterior distribution at the $(k-1)$-th instante is given in (28) with the parameters a^*, b_1^* and b_2^*. Thus, the one-step forecast made at that instant and its error estimate are obtained using the following formulas:

$$\begin{aligned} E^*(X_{k+1}) &= \frac{b_1^*}{1-2a^*} \\ \mathrm{Var}^*(X_{k+1}) &= \frac{b_2^*(1-2a^*) - {b_1^*}^2}{(1-2a^*)^2}. \end{aligned} \tag{32}$$

where Var^* and E^* denote respectively the posterior variance and the expected value when the posterior distribution is obtained by using the Jeffrey's rule.

Example 5.1 In this example the updating of the prior distribution by an arbitrary reassessment occurs after instants 5,10 and 15. Data are generated from the Student-t distribution given in (29) adopting the following parameters: up to time 5, $a = 0.0001, b_1 = 0$ and $b_2 = 0.1$; for times 6 to 10, $a = 0.2, b_1 = 10$ and $b_2 = 167$; the next five data are generated from the Student-t distribution with $a = 0.4, b_1 = 10$ and $b_2 = 501$ and for the last five data $a = 0.01, b_1 = 10$ and $b_2 = 1003$, are assumed.

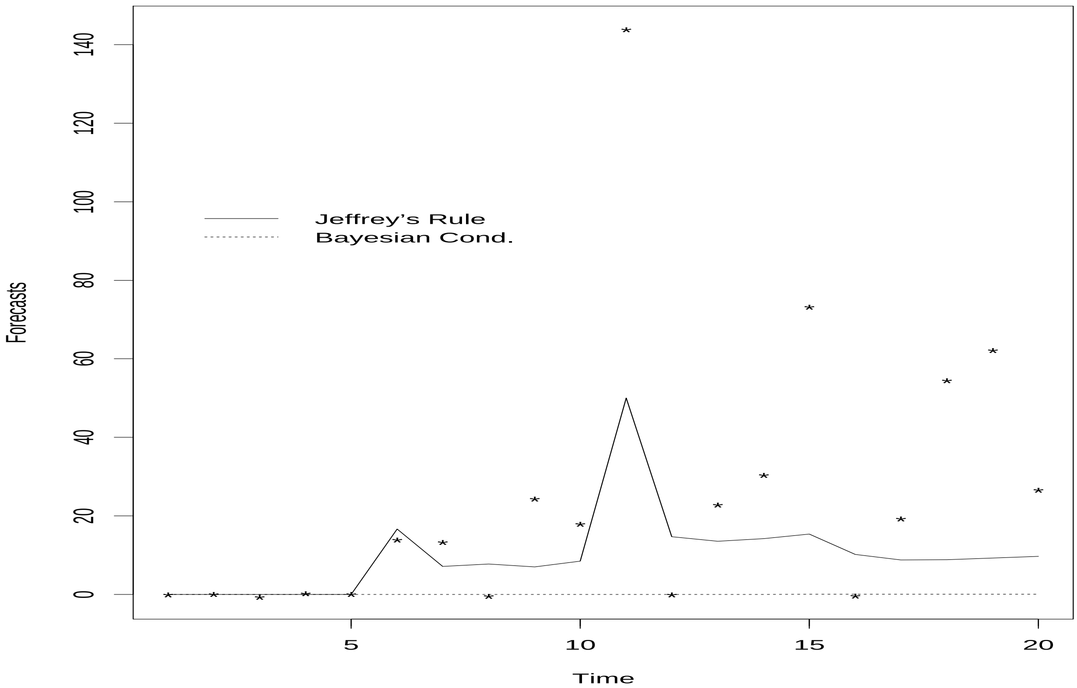
The forecasts and errors obtained by using both procedures are presented in Table 1. Comparing the performances of the Jeffrey's rule and the Bayesian Conditioning, it may be noticed that although the forecasts obtained by using the Jeffrey's rule are, in general, closer to the real values, the errors associated to these forecasts are bigger - what implies the low accuracy of the forecasts.

Table 1: Forecasts and Errors Using the Jeffrey's Rule and the Bayesian Conditioning

	forecast		*Error*	
Real value	Bayes	Jeffrey	Bayes	Jeffrey
0.0937532	0	0	0.1000	0.1000
0.2348605	9.376×10^{-6}	9.376×10^{-6}	0.1000	0.1000
-0.5086271	3.286×10^{-5}	3.286×10^{-5}	0.1000	0.1000
0.4470131	-1.799×10^{-5}	-1.799×10^{-5}	0.1000	0.1000
0.2110936	2.669×10^{-5}	2.669×10^{-5}	0.1000	0.1000
14.0922356	4.779×10^{-5}	16.666	0.1000	0.555
13.4479058	1.456×10^{-3}	7.121	0.1198	64.1297
-0.2593757	2.800×10^{-3}	7.754	0.1379	61.3189
24.4903995	2.774×10^{-3}	7.026	0.1379	61.0515
18.1194724	5.221×10^{-3}	8.481	0.1978	79.2642
144.0349004	7.03×10^{-3}	50.000	0.2306	4.9999
0.1140547	2.142×10^{-2}	14.6987	2.3029	1696.8662
23.0120705	2.143×10^{-2}	13.532	2.3027	1576.7725
30.5514515	2.373×10^{-2}	14.234	2.3552	1466.1387
73.4094281	2.678×10^{-2}	15.360	2.4481	1382.1216
-0.1692146	3.411×10^{-2}	10.204	2.9856	919.3461
19.4263359	3.408×10^{-2}	8.770	2.9853	802.9041
54.6328834	3.602×10^{-2}	8.863	3.0225	796.9011
62.3209464	4.147×10^{-2}	9.258	3.3198	807.935
26.7705502	4.769×10^{-2}	9.711	3.7066	824.8896

Figure 1 also shows that the Jeffrey's rule (full line) is more efficient to the incorporation of unpredictable information. Notice that the Bayesian Conditioning (dotted line) does not incorporate the changes imposed at the instants 5,10 and 15. The forecasts obtained by its use remain almost unchangeable at all steps (data are represented by stars). Despite this the errors are small (see Table 1), what suggests that the Bayesian Conditioning is a good procedure to make forecasts.

Figure 1: Forecasts using the Jeffrey's rule and the Bayesian Conditioning



Example 5.2 The Chilean stock market is an emerging market, what means that it can experience more changes than the developed ones (Duarte and Mendes, 1997), because it is more affected by the political atmosphere. In fact, Maeda (1996) reports that changes in the behavior of the series of Chilean stock market returns are a consequence of the receipt of information which is not previously anticipated, and, if surprises do not occur, the behavior of the returns tends to remain unchangeable. This statement is similar to the one made by Mandelbrot (1963) to efficient markets. Moreover, it is also reasonable to assume that, conditionally on the volatility and in the expected return, returns observed between two shocks are independent, having a common normal distribution (Correa, 1998).

Consequently, the Chilean stock market returns are $O(\mathbf{1})$-invariant and satisfy the condition (27) both before and after the receipt of new information. It is then sufficient to define the new parameters a^*, b_1^* and b_2^* after the occurrence of each shock, what is equivalent to assuming the natural Jeffrey-conjugacy.

Figure 2 shows the behavior of the real returns of Serie ENDESA (represented by *) and the forecasts using both the Jeffrey's rule (full line) and the Bayesian Conditioning(traced line). It can be noticed that the Jeffrey's rule has a greater capability to incorporate changes in the forecast process, what makes this rule more flexible than the Bayesian Conditioning when a shock occurs.

Figure 2: Forecasts Using the Jeffrey's Rule and the Bayesian Conditioning - ENDESA Returns

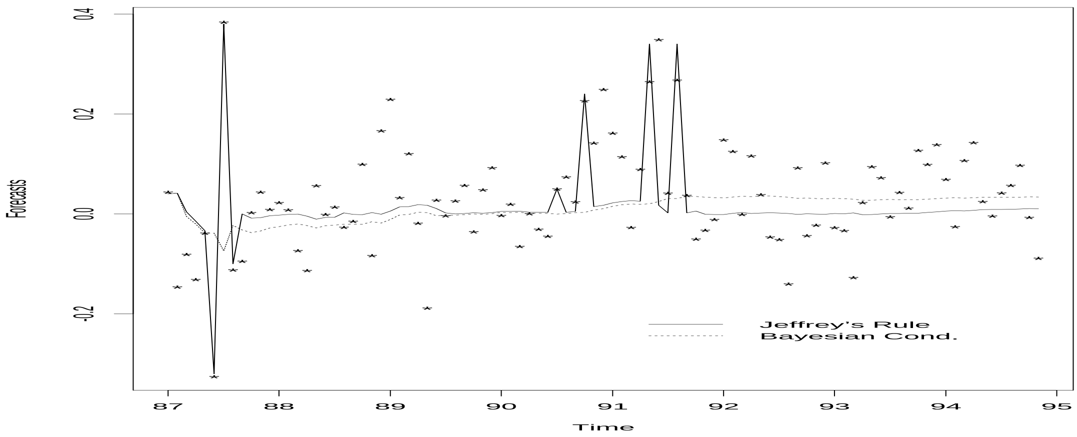
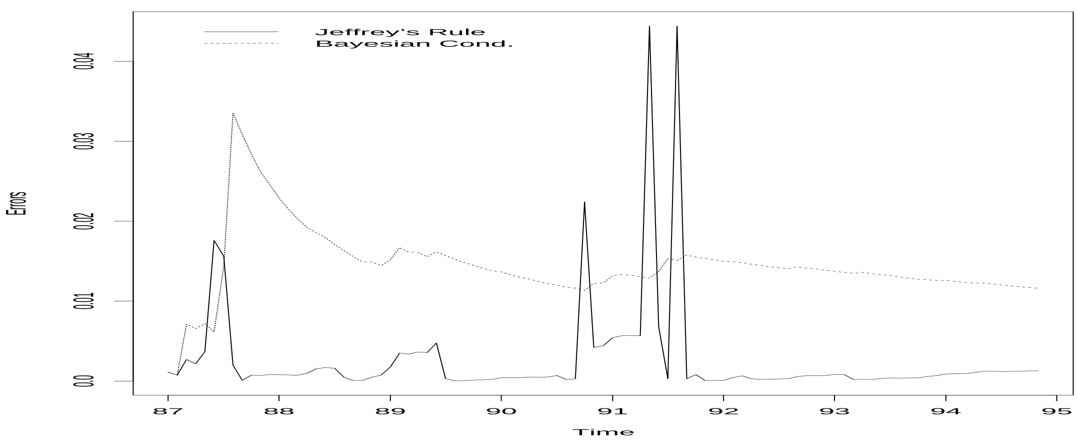


Figure 3: Errors on the Forecasts - ENDESA Returns



Notice from figure 3 that errors on the forecasts made by using the Jeffrey's rule (full line) are smaller and reach their highest values in periods when greater volatility is observed. It means that forecasts given by the Jeffrey's rule are, generally, more accurate.

6 Conclusions

To incorporate unanticipated information in both forecasts and inferences is a common problem in practical situations as well as a real problem in the Bayesian statistics. If it is assumed that posterior distributions are only those found by using the Bayes' formula, new information can only be included in the forecasts if, after the receipt of the information, it is assumed as certain the occurrence of the conditioning event.

In emerging stock markets, for example, the behavior of financial series is commonly affected by international monetary policies, oscillations in the stock markets of influential countries, wars, and so on. In general, the unpredictability behind this kind of information makes the use of the Bayesian Conditioning inappropriate.

The Jeffrey's rule is a possible way to make predictions in these cases. As seen before, this rule is more efficient in the addition of unpredictability, even though it also has some limitations. To use the Jeffrey's rule as a forecasting procedure, the receipt of new information must only lead to partial changes on the prior distribution, as shown in this paper.

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