Ricardo Fonseca Couto

NONPARAMETRIC DEPENDENCE MODELLING FOR SPACE-TIME CLUSTER DETECTION

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UFMG

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Banca Examinadora:
Orientador: Prof. Dr. Luiz Henrique Duczmal
Co-Orientadora: Prof. Dra. Denise Burgarelli Duczmal
Membro: Prof. Dr. André Luiz Fernandes Cançado
Membro: Prof. Dr. Gladston Juliano Prates Moreira
Membro: Prof. Dr. Ricardo Hiroshi Caldeira Takahashi
Membro: Prof. Dr. Roberto da Costa Quinino

Programa de Pós-Graduação
Departmento de Estatística
Universidade Federal de Minas Gerais

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Resumo

Estruturas de dependência são estudadas exaustivamente em diversas aplicações. Nesta tese, uma nova metodologia para a detecção de *clusters*, chamada de Distância Ponderada de Voronoi, é apresentada levando em consideração não somente a localização dos pontos, mas também sua estrutura temporal. Usando o espaço de variáveis ao invés da localização geográfica, como em estatística espacial, e o toro em lugar do plano cartesiano, esta metodologia permite que o usuário aplique a ideia para um número maior de cenários alternativos. Em particular nos mercados financeiros, diferentes modelagens de dependência entre ativos podem levar a mudanças drásticas na alocação de recursos e a diferentes exposições a risco. Além disso, estas relações de dependência podem ser utilizadas como mecanismos para a detecção de crises financeiras mais rápidos que os modelos fundamentalistas, através do efeito contágio. Inicialmente, esta aplicação é realizada entre ativos de um mesmo mercado, o mercado americano, comparando a metodologia proposta com metodologias mencionadas na literatura como coeficientes lineares e copulas. Esta abordagem foi estendida para ativos de mercados distintos, a fim de se analisar a disseminação de crises financeiras entre diferentes mercados. Resultados obtidos através de simulações e aplicações com dados reais mostraram melhorias se comparados com abordagens clássicas, especialmente em períodos financeiros turbulentos.

Palavras-Chave: *Cluster*, azulezamento, algoritmo, Efeito Contágio.

Abstract

The dependence structures have been exhaustively studied in many applications. In this thesis, a new methodology for cluster detection is presented, i.e. the Weighted Voronoi Distance (WVD), taking into consideration not only the location of the points but also their time structure. Using variables space instead of geographical location as in spatial statistics and the torus instead of a regular Cartesian plane, this methodology allows the user to apply the rationale for more alternative scenarios. Particularly in financial markets, different dependence modelling among assets can lead to significant changes in asset allocation and different risk exposures. Besides, these dependence relationships can be used as mechanisms to detect financial crisis more quickly than fundamental models through the contagion effect. Initially, this application is run using assets from the same market, i.e. the US market, comparing the proposed methodology with methodologies mentioned in literature such as linear coefficients and copulas. This approach will be extended to assets from distinct markets in order to analyse the financial crisis dissemination across different markets. Results obtained from simulations and real data applications showed improvements compared to classical approaches especially in during turbulent financial periods.

Keywords: Cluster detection, tessellation, algorithm, Contagion Effect.

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Apresentação

A relação de dependência entre variáveis aleatórias é motivo de estudo há séculos. Inicialmente, medidas de associação lineares foram desenvolvidas e, com o passar do tempo, medidas não lineares foram apresentadas. Paralelamente, os estudos de séries temporais também se tornaram difundidos na análise que incorporam os efeitos do tempo no comportamento de uma variável aleatória. Entretanto, no mundo real, podemos encontrar variáveis aleatórias que estão, de alguma forma associadas, mas também possuem o efeito da variável tempo tanto dentro das próprias séries como entre elas. Como entender este comportamento? É possível verificar de forma rápida e eficiente um aumento ou diminuição da dependência destas variáveis para pequenas variações?

Sendo encontrada em diversas áreas do conhecimento, estas questões apontadas acima são de extrema importância para o entendimento de fenômenos e tomadas de decisão. Por isso, esta tese foi desenvolvida através do estudo de estruturas de dependência entre variáveis aleatórias, culminando na proposta de uma nova medida de dependência. Esta abordagem possui algumas vantagens sobre as abordagens já apresentadas na literatura: (i) possibilita a utilização de dados em tempo real, (ii) intrinsecamente traz a estrutura temporal das séries, (iii) se baseia em técnicas de estatística espacial próprias para a detecccção de *clusters*, definidos formalmente no Capítulo 1 e (iv) utiliza uma topologia capaz de suportar uma variedade maior de modelos alternativos.

Esta tese está organizada da seguinte forma: no Capítulo 1, uma apresentação e contextualização do problema são expostas, incluindo uma descrição detalhada dos trabalhos realizados nesta área. Posteriormente, no Capítulo 2, a metodologia de Cópulas é apresentada e as famílias que serão utilizadas neste trabalho são descritas. No capítulo seguinte, Capítulo 3, as metodologias de estatística espacial presentes na literatura são apresentadas e a nova metodologia, principal interesse desta tese, é apresentada detalhadamente. No Capítulo 4, simulações utilizando a nova metodologia são expostas e comparadas com métodos tradicionais e no Capítulo 5 a metodologia é aplicada em dados reais e também comparada com a abordagem de cópulas. Por fim, no Capítulo 6 esta tese é concluída e as considerações finais são realizadas.

Chapter 1

Introduction

Dependence between two variables has been of major concern in many fields not only for statisticians but also for both researchers and practitioners from biological to social sciences. Along the last centuries, independently of any particular application, many measures have been proposed. Although the linear correlation coefficient (Pearson's ρ) proposed by Pearson (1895) is the most popular, other concordance measures such as rank correlation have been introduced to better apprehend correlation drawbacks. Spearman (1904) proposed a concordance measure (as known as "Spearman's ρ") which does not assume linear relationships among variables but only monotonicity. Kendall (1938) has also proposed a concordance measure, the "Kendal's τ", with similar characteristics and assumptions. Pantaleo et al. (2011), for instance, show a comparative study of nine covariance matrix estimators and Deng and Tsui (2013) propose a different method to estimate the covariance matrix using matrix-logarithm transformation, to cite only a few. One of the main issues about the covariance matrix is due to its non-static nature. Many studies discuss the dynamic behaviour of the covariance matrix and provide alternative methods and metrics to estimate and evaluate the change taking into account the time structure (Andersen et al., 2009; Bauwens et al., 2006; Engle, 2002)

Introducing the idea of decomposing multivariate distributions behaviour into functions of the marginal distributions of the random variables and of their dependence, Sklar (1959) named "copulas" the function that absorbs all the dependence structure. These approaches have some equivalence among each other, although the transformations are not always possible to obtain in closed analytical form. Since then, copulas have been used in many fields of science from geophysics to actuarial sciences and finance (Cherubini et al., 2004; Nelsen, 2006; Patton, 2012; Salvadori et al., 2007).

However, data time structure such as autocorrelation and homoskedasticity were not taken into consideration in these studies. For time series analysis, other tools have been developed separately such as Auto Regressive Conditional Heteroscedasticity/Generalized Auto Regressive Conditional Heteroskedasticity (ARCH/GARCH) developed by Engle (1982) and Bollerslev (1986), and its derivations, for instance, Asymmetric Power ARCH (Ding et al., 1993), Exponential GARCH (Nelson, 1991), and Multivariate GARCH (Bauwens et al., 2006).

Recently, many studies have been published using copulas and conditional volatility to explain the contagion effect in financial markets (Brunnermeier and Perdersen, 2005; Forbes and Rigobon, 2002; Kaminsky et al., 2003; Naoui et al., 2010; Rodriguez, 2007). Particularly in financial applications, the *contagion effect* is when the returns of financial assets show increase in their dependence structure or, graphically speaking, appear in a unlikely manner around the regression line. An important aspect of this phenomenon is that the high positive returns can also be correlated with high negative returns as in financial "bubbles". In quantitative finance, both risk management and asset allocation procedures rely strongly upon dependence measures to estimate risk factors and achieve efficient diversification. As stock returns show asymmetric lower and upper tail dependence, the behaviour cannot be captured by traditional linear correlation coefficient (Ang and Bekaert, 2002; Granger and Silvapulle, 2001; Login and Solnik, 2001).

Copula functions also turn out to play an important role in dependence modelling especially due to concordance characteristics, capacity to model tail dependence and flexibility to work in a non-Gaussian world. However, in order to use the Extreme Value Theory (EVT) and Generalized Pareto (GP) distribution model for both left and right tails separately, the independence and identically distribution (i.i.d) assumptions cannot be broken. Therefore, an ARMA/GARCH approach is usually applied to generate the filtered conditional residuals which can be assumed to be independent and identically distributed (Ghrobel and Trabelsi, 2009; Nystrom and Skoglund, 2002). Besides, copula selection and validation tools are not unanimous. For instance, Huard et al. (2006) considered copulas selection without counting for marginal modelling. Silva and Lopes (2008) show advantages of estimating all parameters and used the deviance information criterion, proposed by Spiegelhalter et al. (2002). Michiels and Schepper (2013) (and references therein) show different graphical methods to visualize the fit.

For some other knowledge areas, particular tools were developed to take into consideration the phenomena specificity. In epidemiology, for instance, for cluster detection applications in spatial statistics, many algorithms have been proposed.

Formally, a *spatial cluster* is the localized portion of the domain which contains a higher-than-average proportion of cases over controls, and a *space-time cluster* can be defined as unexpected concentrations of cases in a time series sequence of maps. One of the predecessors in this field was Kulldorff (1997) who developed a spatial scan statistics to detect a geographical cluster as early as possible. Thereafter, Kulldorff (2001) proposed a prospective time scan method improved by Duczmal et al. (2006). Other methods such as Conley et al. (2005), Wieland et al. (2007), Yiannakoulias et al. (2007) and Duczmal et al. (2008) have also been presented to detect clusters using different techniques. Duczmal et al. (2011) showed an extension for a prospective space-time scan called the Voronoi Based Scan which uses the spatial tessellation and Voronoi cells boundaries to detect clusters.

Although, at a first glance, these problems seem to be unrelated, the rationale behind them is quite similar. Suppose X, Y are two random variables with an unknown dependence structure between them. Without assuming any condition for the relationship such as linearity or lack of sample autocorrelation, the main interest is to analyse the time behaviour of the dependence structure which can be used for many applications such as cluster detection in spatial statistics and contagion effect signalling in financial markets. As described in Dornbusch et al. (2000), the financial contagion detection can serve as a tool to anticipate a crisis in financial markets.

This work proposes a new non-parametric methodology based upon a non-ordinary metric: the Weighted Voronoi Distance (WVD). Although extensive theoretical discussion can be found in Okabe et al. (2000) about spatial tessellation which has been widely used in many fields such as biology and physics, it has not been used, to the best of our knowledge, to model the space-time dependence structure between two random variables. The main goal is to use the technique to develop a method able to detect the increase in dependence more quickly than traditional measures. To illustrate the potential advantages, simulations were run and real financial datasets were used to verify how quick the 2007 subprime crisis could have been warned.

This thesis is organized as follows. Chapter 2 describes the copula methodology for dependence modelling. Chapter 3 defines the new methodology of cluster detection describing the inference procedure and the space-time scan. Chapter 4 presents simulations run for different scenarios. The following chapter, Chapter 5 describes the financial markets behaviour and the possibility to apply the methodology in this field using real datasets and conclusions are pointed out in Chapter 6.

Chapter 2

Dependence Modelling using Copulas

Copulas are the cutting edge technique to analyse the dependence structure today due to its characteristics and flexibility (Boubaker and Sghaier, 2013; Meucci, 2011; Patton, 2012). Therefore, this Chapter defines this measure in section 2.1, mentioning the practical issues and benefits of such mathematical tool.

Broadly speaking, two parametric copula families are the most common and mentioned in literature, i.e. the elliptical and Archimedean copulas. In this work, the elliptical family will be described and used as parameters interpretation is eased through the transformation to traditional ρ. Furthermore, for Archimedean families, not only are such analytical transformations not possible making it harder to interpret the parameters, but they also have showed poor results for Goodness-of-fit test for financial series (Patton, 2012).

2.1 Copulas

The word *copula* has its origin in Latin meaning "bond" or "tie". In statistics, Sklar (1959) was the first to mention this particular word in a theorem which bears his name (pointed out in this section). Since then, this measure has become of great interest among statisticians especially due to its scale-free characteristics and the possibility of constructing families of bivariate distributions.

Formally, let $I = [0, 1]$. Then, copulas can be defined as:

Definition 2.1.1 *A two-dimensional copula is a function* $C{:}I^2 \to I$ *such that:*

(i) $C(0, x) = C(x, 0) = 0$ *and* $C(1, x) = C(x, 1) = x, \forall x \in I$*;*

(ii) for all $a, b, c, d \in I$, with $a \leq b$ and $c \leq d$,

$$V_c([a,b]x[c,d]) = C(b,d) - C(a,d) - C(b,c) + C(a,c) \leq 0. \quad (2.1)$$

The function V_c is also called the $C - volume$ of the rectangle $[a,b] \times [c,d]$. Equivalently, a copula is a bivariate distribution whose margins are uniform in $[0,1]$ restricted to the unit square $[0,1] \times [0,1]$.

The importance of copulas in statistical field has been increasing due to Sklar's Theorem.

Theorem 2.1.1 *(Sklar's Theorem) Let G be a two-dimensional distribution function with marginal distribution functions F_1 and F_2. Then there exists a copula C such that $G(x,y) = C(F_1(x), F_2(y))$. Conversely, for any distribution functions F_1 and F_2 and any copula C, the function G defined above is a two-dimensional distribution function with marginals F_1 and F_2. Furthermore, if F_1 and F_2 are continuous, C is unique.*

According to Sklar's theorem, using a collection of copulas it is possible to construct bivariate distributions with arbitrary margins. Nelsen (2006) and references therein describe in details many parametric families and their characteristics. Normal and t copulas and the Farlie-Gumbel-Morgenstern (FGM) copula family are widely used in literature as mentioned by Aas (2004); Kolev and Paiva (2009); Manner and Reznikova (2012); Patton (2012) to mention only a few. However, the FGM family can only model relatively weak dependence (Nelsen, 2006). Thus, the immediate interest is to focus on Normal and t copulas.

2.1.1 Elliptical Copulas

Let Φ_ρ be the standard bivariate normal joint distribution with correlation coefficient ρ. Then, the Normal (or Gaussian) copula is given by:

$$C_\rho^N(u,v) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v)) \quad (2.2)$$

where Φ^{-1} denotes the standard normal distribution function. It is worth mentioning that since there is no analytical expression for Φ^{-1}, Φ_ρ has also no closed form.

The t (or Student) copula can be defined analogous to the normal copula. Let $T_{v,\rho}$ be cumulative bivariate $t_{v,\rho}$ distribution with correlation coefficient ρ. Then, the t-copula is given by:

$$C_{v,\rho}^T(u,v) = T_{v,\rho}(T^{-1}(u), T^{-1}(v)) \quad (2.3)$$

Although both copulas have similarities, their behaviour in case of extreme events (when the interest is to see the joint extremes) differ substantially. In order to study the tail behaviour, the upper and lower tail dependences (λ_U and λ_L, respectively) can be defined as:

$$\begin{aligned}\lambda_U &= \lim_{t \nearrow 1} P(Y > F_2^-(t) | X > F_1^-(t)) \\ &= 2 - \lim_{t \nearrow 1} \frac{1 - C(t,t)}{1-t} \\ \lambda_L &= \lim_{t \searrow 0} P(Y \leq F_2^-(t) | X \leq F_1^-(t)) \\ &= \lim_{t \searrow 0} \frac{C(t,t)}{t}\end{aligned} \tag{2.4}$$

It is possible to show that the Gaussian copula has 0 tail dependence for any given correlation ρ. However, t-copula has its upper and lower tail dependence evaluated by:

$$\lambda_{v,\rho}^t = 2t_{v+1}\left(\frac{-[(1+v)(1-\rho)]^{\frac{1}{2}}}{(1+\rho)^{\frac{1}{2}}}\right) \tag{2.5}$$

This characteristic is important when the phenomena studied present extreme events. As the methodology application with real datasets showed in Chapter 5 is one of these cases, the Student copula will be applied to both simulation studies in Chapter 4 and case study in Chapter 5.

Chapter 3

Dependence Modelling Measures and Cluster Analysis and Detection

In this Chapter, the main contribution of this thesis is presented. First of all, the new measure, called Weighted Voronoi Distance, is defined followed by a cluster detection algorithm and a Space-Time Scan based on it.

3.1 Cluster Analysis

3.1.1 Weighted Voronoi Distance

The first step to build the proposed metric is to define Voronoi diagram. Consider n point in the space domain and the set $P = \{(x_i, y_i) : i = 1, ..., n\} \subset \Re^2$. For $i = 1, ..., n$, the *Voronoi cell* $v(i)$ consists of those points in $\Re^2$ which are closer to (x_i, y_i) than any other point in P. The *Voronoi diagram* is the collection of cells $v(i), i = 1, ...n$.

Let v_k be a Voronoi cell which is crossed by the line segment joining points $c_i = (x_i, y_i)$ and $c_j = (x_j, y_j)$, d_k the length of the segment that is in v_k, and a_k the area of the cell v_k. Then, the Weighted Voronoi Distance (WVD) is defined as:

$$WVD_{c_i,c_j} = \sum_{k=1}^{n_k}(d_k/a_k) = \sum_{k=1}^{n_k}(w_k * d_k) \qquad (3.1)$$

where n_k is the total number of cells crossed by the line segment and $w_k = 1/a_i$ is the weight assigned to each cell.

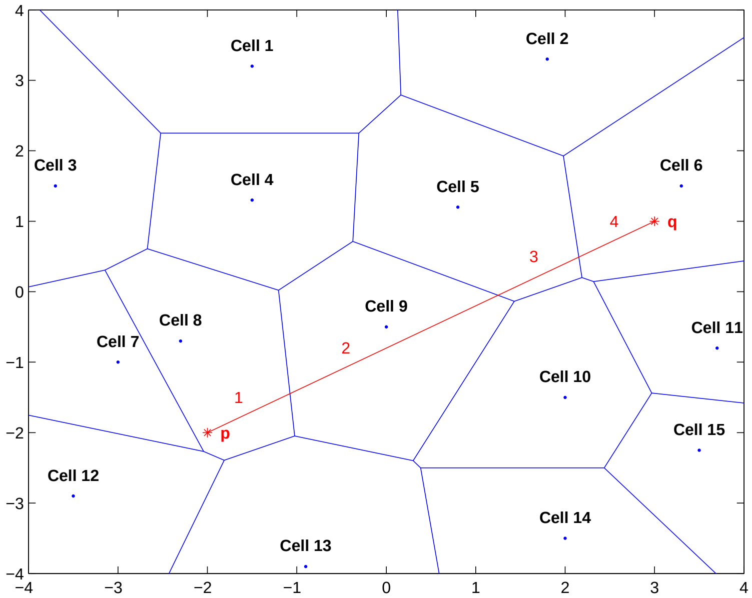


Fig. 3.1 Example of Weighted Voronoi Distance (WVD).

Example 3.1.1 *Let p and q be two points in the plane illustrated in Figure 3.1. The formula becomes:*

$$WVD_{p,q} = \sum_{i=1}^{4} d_i/a_i = d_1/a_1 + d_2/a_2 + d_3/a_3 + d_4/a_4 \tag{3.2}$$

The numbered segments, d_i, emphasize that each segment is weighted by its Voronoi cell area a_i.

3.1.2 Cluster Detection and WVD Space-Time Scan

Having defined the WVD, it is possible to build a cluster detection procedure. Firstly, a data set called *training set (T)* (known as "controls" in some fields) is chosen, the Voronoi diagram is calculated and plotted with these points and the WVD is calculated for all possible pairs. Notably, as the metric is symmetric only $(n^2 - n)/2$ distances have to be calculated, reducing the complexity of the algorithm. Besides, this training set T has to be chosen to represent a randomly distributed population. Secondly, a Monte Carlo simulation is run m times to build the empirical distribution of WVD's. Then, the sum of all WVD in time sequence from a data set called *candidate set (C)* (known as "cases" in some fields) is calculated and compared to the empirical distribution. Given the significance level, it is possible to test the existence of a possible cluster, as defined in Chapter 1, through an one-tail hypothesis test. The procedure can be summarized as:

1. Choose the Training Set (T) which describes the randomly distributed population and calculate the Voronoi Diagram for these points.
2. Calculate the Weighted Voronoi Distance (WVD) for all pairs in the Training Set (T).
3. Build an Empirical Distribution for the WVD using Monte Carlo simulations.
4. Calculate the Weighted Voronoi Distance for the Candidate Set (C).
5. Compare the WVD of the Candidate Set with the appropriate percentile given the confidence level.

To visualize both the Training Set (T) and the Candidate Set (C), Figure 3.2 shows an example of a Training Set with 10 points and a Candidate Set with 3 elements.

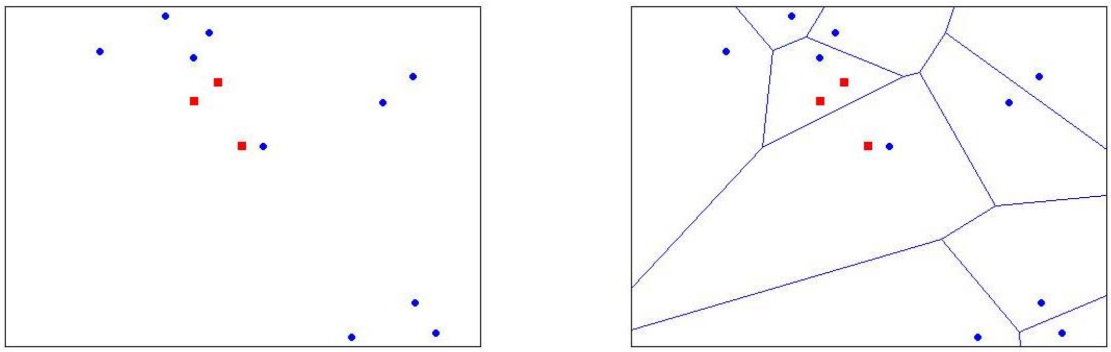


Fig. 3.2 The circles are the training set. The squares are the candidates for a cluster.

It is worth mentioning that the points can hardly be identified as significantly concentrated just looking at the map.

Formally, let c_i be the $i-th$ point, the training set $T = \bigcup_{i=1}^{n_T} c_i$, and n_c be the length of the candidate set C. Then, the WVD_i of a subset of $T = c_{(i-n_c)}, ..., c_i$ is defined as:

$$WVD_i = \sum_{t=i-n_c+1}^{i} WVD_{t,t-1} \tag{3.3}$$

And the Empirical Distribution (ED) for a particular time length (or horizon) n_c obtained running m Monte Carlo simulations is defined as:

$$ED = \bigcup_{i=1}^{m} WVD_i \tag{3.4}$$

Having set the significance level, α, the one-tail hypothesis test is run:

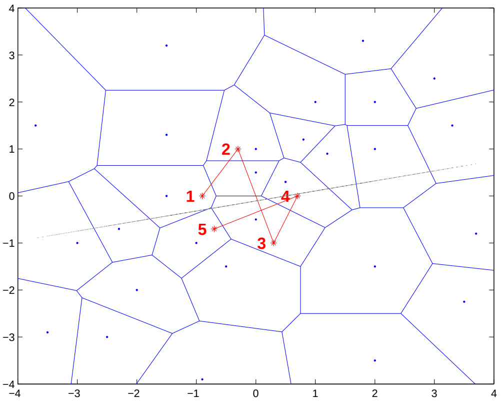
$$\begin{cases} H_0 : WVD_i \geq 100 * \alpha^{th} percentile(ED) \\ H_1 : WVD_i < 100 * \alpha^{th} percentile(ED) \end{cases}$$

One of the advantages of this new metric is that as it has been built upon the real line, it allows no multiple solutions, a problem faced by Duczmal et al. (2011).

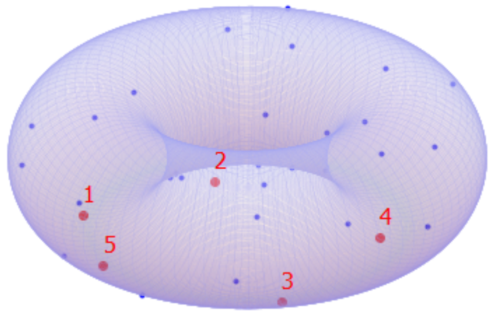
Particularly in financial applications, the contagion effect can be seen when the returns of financial assets increase their correlation getting closer to the regression line. As high positive returns can be followed by high negative returns as in financial unstable periods, instead of applying the WVD methodology for the original plane, i.e. returns of asset y against returns of asset x, a torus is created with its external longitudinal axis being the regression line of the original plane, maintaining the returns dependence characteristics. Furthermore, this geometry construction extinguishes the border problem when dealing with Voronoi Diagrams in classical Cartesian plane as borders are connected and no infinite border is necessary.

In Figure 3.3 the original returns series are plotted for three different situations: a centered cluster, no cluster, and a border cluster.

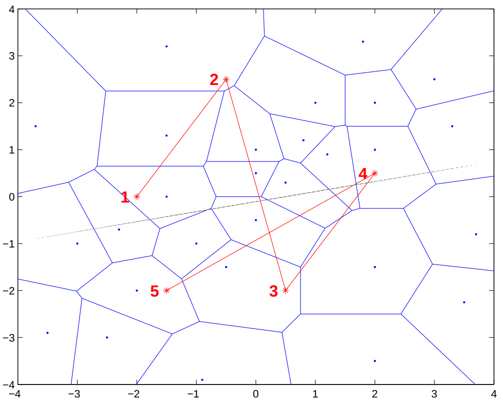
The first two cases are visually straightforward to recognize. However, the latter case is only obtained due to the torus construction. Otherwise, should the points be connected through the middle of the map instead of through the borders, the Voronoi Distance could be high enough not to detect the financial movement. Although the distances are actually calculated in the Cartesian plane using replicas around the original map, the geometrical visualization of the phenomena with torus is easier especially when high and low returns are correlated.



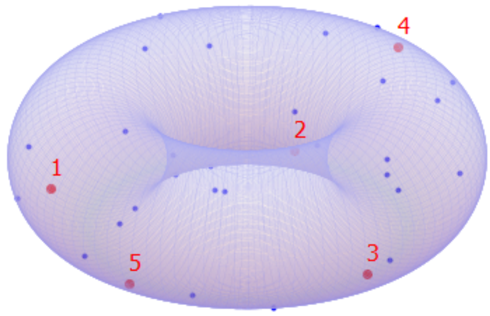
(a) Centered Cluster in the Cartesian plane



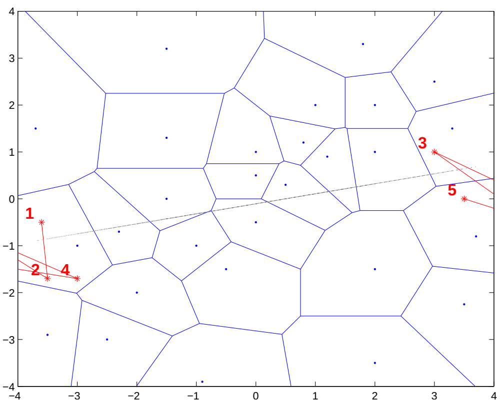
(b) Centered Cluster represented in a torus



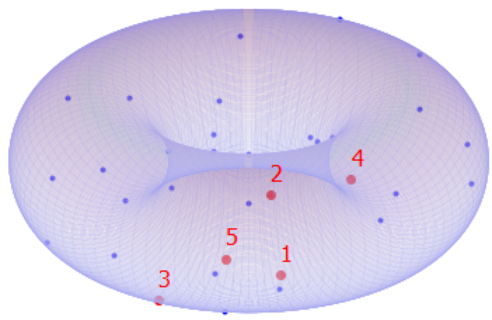
(c) No Cluster in the Cartesian plane



(d) No Cluster represented in a torus



(e) Border Cluster in the Cartesian plane



(f) Border Cluster represented in a torus

Fig. 3.3 The numbered dots represent the cluster candidates considering their order of occurrence. Regression lines are plotted in the Cartesian planes representation.

Chapter 4

Simulation Study

After describing the new methodology in Chapter 3, this chapter shows results and discussion about the methodology simulations under different scenarios, and comparison with copula approach.

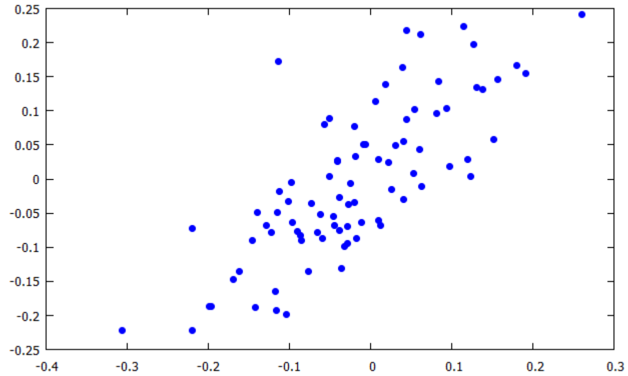
For both methods, a training set (T) assumed to be the null hypothesis of 504 observations was generated from a bivariate Normal distribution with zero mean, 0.10 standard deviation for each variable, and 0.70 correlation coefficient between them. Formally, the empirical distribution is given by:

$$X \overset{H_0}{\sim} N(\mu, \Sigma), \tag{4.1}$$

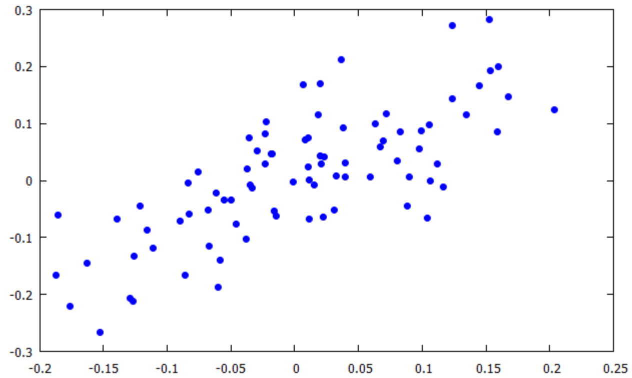
where $\mu = [0, 0]$ and $\Sigma = \left[\begin{smallmatrix} 0.010 & 0.007 \\ 0.007 & 0.010 \end{smallmatrix}\right]$.

As alternative scenarios, three different changes in behaviour were tested. Firstly, the mean was kept at zero and the correlation was increased to 0.85. This scenario simulates the simple increase in correlation between two random variables. The second scenario was generated from distribution with the same covariance as the null hypothesis, but the mean for each sample was either $[0.04, 0.04]$ or $[-0.04, -0.04]$. The decision as taken from a Bernoulli Distribution with p parameter equals to 0.5. Thus, keeping the correlation constant, this scenario is expected to verify the detection capacity of the proposed algorithm in a situation where regular tools usually do not work as well as in the first scenario. Finally, the third scenario was generated changing the mean vector to $\mu = [-0.04, -0.04]$. These cases were especially chosen due to practical applications were this phenomena are found such as financial markets (described in details in Chapter 5). The alternative scenarios are shown in Figure 4.1.

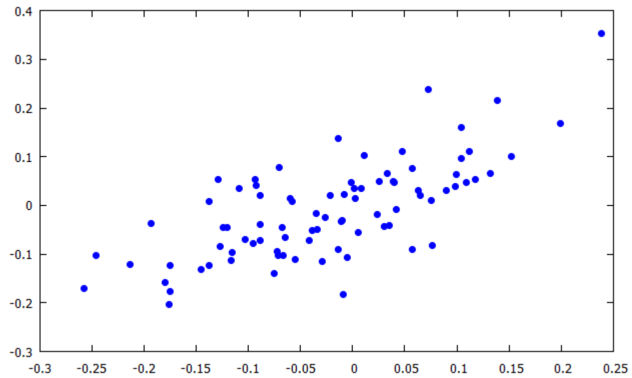
As discussed by Kulldorff (2001), purely repeating spatial analysis cannot be adequate to detect emerging clusters. Should the emerging cluster have occurred in the



(a) Increase in correlation to 0.85.



(b) Alternating points between mean 0.04 and -0.04.



(c) Translated points around (-0.04,-0.04)

Fig. 4.1 Scatter plot for alternative scenarios tested.

last few observations, purely spatial analysis for the whole period dilutes the cluster strength and diminishes the power to detect such cluster. Conversely, for short periods, adjustments for multiple testing should be made when repeated analysis every month are taking place. Therefore, simulations were run using four different time periods, i.e. $n = 21$, 42, 63 and 84 points, and tests were made after each step to check possible cluster detections.

For each horizon, the WVD Space-Time Scan compares the Voronoi Distance with an empirical distribution as defined in equation 3.4. In each simulation, a collection of n points was randomly selected from the 504 original points and Voronoi tessellation, respecting their order, and the total Weighted Voronoi Distance was recorded. This step was repeated 100,000 times to build the empirical distributions for each horizon.

With the empirical distributions, it was possible to calculate the critical values for each horizon and for different significance levels: 0.5%, 1%, 2%, 5%, 10%, 20%, 50%, and 80%. Table 4.2 describes these values which play an important role in the proposed test.

Table 4.1 Critical Values for the Empirical Distributions for different horizons.

Significance Level	Horizons			
	21	42	63	84
0.5%	7213.7	17144.7	27693.7	37634.0
1.0%	7596.2	17711.2	28333.2	38855.2
2.0%	8046.1	18333.3	29157.6	39818.6
5.0%	8662.4	19229.8	30165.1	41026.8
10.0%	9220.7	20027.4	31128.5	42195.4
20.0%	9868.7	20979.4	32314.3	43614.0
50.0%	11127.4	22863.4	34590.8	46306.2
80.0%	12433.5	24682.9	36933.2	48999.4

In order to calculate both the cluster *Power of Detection* and *the Average Detection Delay*, 10,080 calculations for Copula and WVD Space-Time Scan approaches were made using the same datasets. The "Power of Detection" column shows the proportion of the simulations that detected a significant cluster and the "Average Detection Delay" represents the mean of the time until detection given that the method detected the cluster. The results can be seen in Table 4.2. The scenario 1 is the increase in correlation, scenario 2 is the alternating points and scenario 3 is the translated points following the order in Figure 4.1.

Table 4.2 Simulation results for different time horizons (95% confidence level).

	Methodology	Number of Points	Power of Detection	Avg. Detection Delay
Scenario 1	**Copula**	21	0.92	7.16
		42	0.95	19.15
		63	0.98	31.11
		84	1.00	45.44
	WVD Scan	21	0.77	11.10
		42	0.99	21.38
		63	0.99	25.66
		84	1.00	26.73
Scenario 2	**Copula**	21	0.78	9.61
		42	0.77	22.27
		63	0.77	35.70
		84	0.78	49.89
	WVD Scan	21	0.79	11.21
		42	0.99	21.01
		63	1.00	25.00
		84	1.00	25.92
Scenario 3	**Copula**	21	0.71	8.46
		42	0.67	19.60
		63	0.67	32.20
		84	0.67	44.94
	WVD Scan	21	0.79	11.33
		42	0.99	20.89
		63	1.00	24.85
		84	1.00	25.87

As results were similar for all other significance levels, the analysis will be made for the 95% confidence level and tables for the remaining levels can be found in the Appendix.

4.1 Discussion

Having proposed the new methodology for cluster detection and run simulations for three different alternative scenarios for both Copulas and WVD Space-Time Scan, it is possible to compare both methodologies.

First of all, both methods behaved similarly for the scenario where the increase in dependence occurs through an increase in off-diagonal terms of the covariance matrix. Such a situation was expected for copulas as a direct transformation can be made to work with the linear correlation coefficient in question. On the other hand, for the same scenario, the WVD approach was also able to capture the data structure change and signalize the increase in dependence.

Nonetheless, in situations where the increase in dependence happens in not an ordinary way and cannot be easily seen in the scatter plot as in Figure 4.1, the proposed methodology showed better results for both Power of Detection and Average

Detection Delay than Copula. By construction, copulas (and the linear correlation coefficient) are not able to capture changes in mean nor changes in behaviour when data oscillates in extremes but do not move away from the original regression line. Through the parameters space tessellation and the use of the torus geometry, the proposed methodology could perform better than copulas to detect and signalize these changes. Although changes around the regression line are common and important, they are not the only type of dependence change that can happen between any two random variables.

Chapter 5

Financial Markets and the Weighted Voronoi Distance Space-Time Scan

This chapter describes the behaviour of global financial markets during crisis (section 5.1 and applies the methodology proposed in Chapter 4 to detect clusters in financial assets returns. A comparison with copula methodology described in Chapter 3 is made and the results are shown in sections 5.2 and 5.3.

5.1 Financial Market Crisis

Financial markets often crashes. Although investors may face different types of risks, the risk of collapse in particular is usually one of the most important risks, especially due to its impact in investor's portfolio. In October 1987, for example, the stock market declined by over 20 percent in the US and Canada and over 25 percent in the UK. The "black Monday" is the largest one-day percentage decline ever recorded in the US stock market. Ten years after, in 1997, the Asian financial crisis caused the devaluation of many Asian currencies, for instance the Thai baht, the South Korean won and the Indonesian rupiah which declined over 80 percent in one year period. Rapidly, it spread out to developed countries. The 2007 crisis with its roots in subprime housing bubble in the United States made the DJIA index plunge over 50 percent in 17 months. These times are crucial for both investors and regulators because the diversification mechanisms usually disappear.

Traditionally, it is believed that continuous long-term deterioration of macroeconomic factors would lead to financial crisis. Krugman (1979) is one of the predecessors in this field having studied how governments should not be able to sustain a pegged exchange rate regime using their foreign reserves to offset markets movements. Otherwise, a "crisis" would be created in the balance of payments. Obstfeld (1986) innovated studying the self-fulfilling balance of payments crises using stochastic models to link macroeconomic variables to the phenomenon. For decades, the early warning systems (EWS) for financial crisis were based on macroeconomic fundamentals and were designed to monitor and predict the events in the mid- to long term, i.e. at least one year. However, due to lack or delay of information and relative low frequency of data releases, these models tend to respond slowly to market changes, although financial data demonstrate high variability.

Recently, many studies have been published trying to analyse and explain the spillover phenomena (Bae et al., 2003; Dungey et al., 2003; Forbes and Rigobon, 1999; Kaminsky et al., 2003; Valdés, 1996). Dornbusch et al. (2000) and Pericoli and Sbracia (2003), for instance, divided the transmission mechanisms into two different categories, i.e. interdependence and contagion. The first category is due to normal interdependence such as geographical position and trade links. This market interaction is based upon fundamentals and is responsible for the comovements in both tranquil and turbulent times. Calvo and Reinhart (2003) named this effect as "fundamentals-based contagion". The second category is another type of dependence which can be seen only in turbulent periods. Dornbusch et al. (2000) argues that this "irrational" phenomenon is based on panic, loss of confidence and herd behavior, but not on financial nor macroeconomic variables. Kyle and Xiong (2001) and references therein discuss how effects other than fundamentals can and actually interfere in market behaviors.

The financial literature describes many mechanisms through which the contagion can be disseminated (Longstaff, 2010, and references therein). First, the negative shocks in one market is associated with negative news that is directly linked to securities, for example, cash-flow and/or collateral, in other markets. Kaminsky et al. (2003); King and Wadhwani (1990); Kiyotaki and Moore (2002) are only a few authors who discuss these market news absorption from different perspectives. Second, Allen and Gale (2000); Brunnermeier and Pedersen (2009) and others analyze how investors reduce the market overall liquidity after a shock. This behavior can be caused by forced liquidation of leveraged positions or investors' ability to obtain funding. Third, the risk premia required by investors is not constant. Therefore, negative shocks in any market can affect the dynamic behavior of the risk premia as investors change their willingness to bear risk. Acharya and Pedersen (2005); Longstaff (2004); Vayanos (2004) show how distressed securities may be predictive of subsequent negative returns in other securities, leading to increase in risk premia asked by investors.

Econometric models have been developed to describe and forecast the economic phenomenon in question. Eichengreen et al. (1996) and Frankel and Rose (1996) showed almost simultaneously how currency crisis can be estimated using logit/probit models. Although they used different input variables and different definitions for "crisis", the rationale is still very similar. Hawkins and Klau (2000) built "vulnerability and pressure indices" based on representative variables presented in the literature and achieved satisfactory results. Edwards (1998) studied the role interest rates play in crisis contagion in open economies using the generalized autoregressive conditionally heteroskedastic (GARCH) approach. The contagion effect among East Asian countries was modeled by Khalid and Kawai (2003) using vector autoregressive (VAR), and no strong support for contagion was found for the period and markets included. Longstaff (2010) used the same approach for ABX subprime indexes and found strong evidence of contagion in the American financial market. Another leading modelling category is the non-parametric approach which utilizes available daily financial data to improve crises detection capability. Kaminsky et al. (1997) popularized the idea of defining threshold values beyond which a crisis would be said to take place using equity prices as one of the indicators. Thereafter, many other studies have been conducted in this line and moderately success has been obtained (Frankel and Saravelos, 2010), although their focus were not crisis prediction but rather an *ex-post* analysis.

In order to study the proposed WVD Space-Time scan methodology performance to signalize 2007-8 financial crisis behaviour and the its presence, two scenarios were analysed: the contagion/spilover effect within the US financial market and the effect in the American, the British and the Japanese markets. The former case is shown in section 5.2 and the latter case is described in section 5.3. This study focus on the three of the most important asset classes, named fixed income, equities, and currency. The benefits of these choices are clear: (i) using daily data instead of macroeconomic variables increases the possibility of detecting a crisis earlier, and (ii) these assets classes represent classes used for diversification benefits in asset allocation procedures.

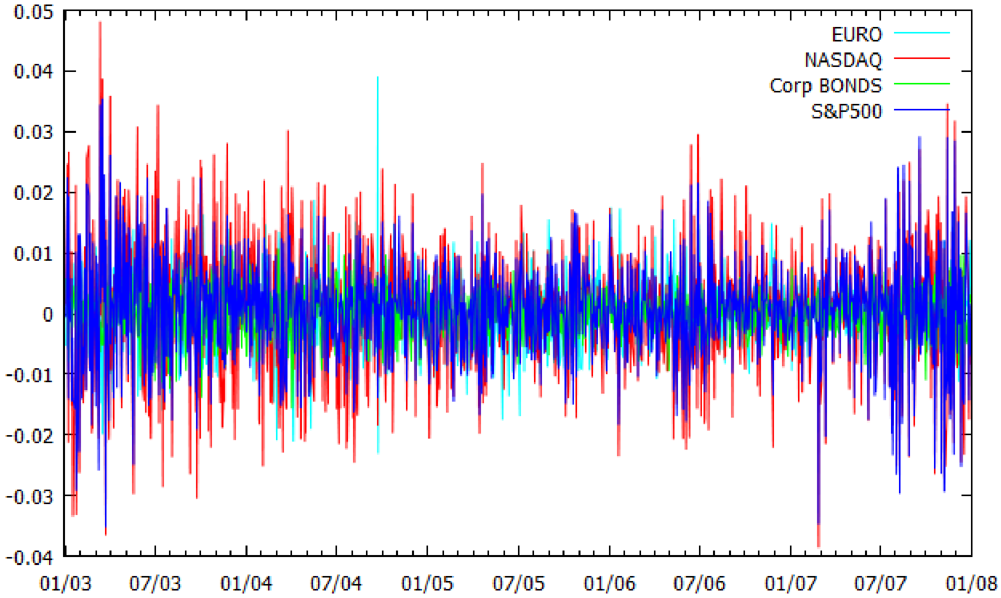


Fig. 5.1 Daily Returns for all 4 asset series between 01/03/2003 and 12/31/2007.

5.2 The US Financial Market Crisis and the WVD Space-Time Scan

The analysis of the American financial market was based upon daily returns of EURO, NASDAQ, iShares iBoxx Investment Grade Corporate Bonds and S&P500 were used from January, 3^{th}, 2003 until December, 12^{th}, 2007, totalling 1253 samples for each series. The data was downloaded from Yahoo! Finance (finance.yahoo.com) and the returns are plotted in Figure 5.1.

It was not until August, 2007, that the Federal Reserve officially published the turmoil in financial markets (FOMC, 2007). Although authorities came to public only in the third quarter of 2007, investors and other market participants have already reacted to and priced the upcoming crisis, changing the behaviour and consequently the dependence structure between assets.

Figure 5.2 shows on the left hand side the scatter plot for the returns around the x-axis, i.e. the returns rotated by the regression line angle. On the right hand side, the respective torus representation is shown. As similar results were obtained for all four pairs of assets, the illustration below is just for one particular case (Corporate Bonds and S&P500).

As discussed in Chapter 4, different periods should be chosen in order to detect clusters. Similar to what was shown by Kulldorff (2001), here a time periodic surveil-

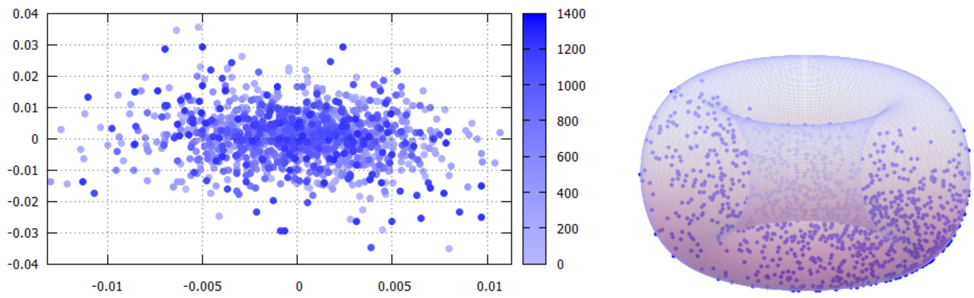


Fig. 5.2 Scatter plot and the torus representation for Corporate Bonds and S&P500.

Table 5.1 P-values for three different dates and four horizons.

	Horizon	12/06	06/07	10/07
Corporate Bonds/S&P	1 month	**0.04**	0.97	**0.00**
	2 months	**0.00**	0.93	**0.00**
	3 months	0.19	0.87	**0.00**
	4 months	0.12	**0.00**	**0.00**
Euro/Nasdaq	1 month	0.40	0.90	**0.02**
	2 months	**0.04**	0.87	**0.00**
	3 months	0.11	0.96	**0.04**
	4 months	0.54	**0.00**	0.06
Euro/Corporate Bonds	1 month	0.08	0.50	**0.02**
	2 months	**0.04**	0.74	**0.00**
	3 months	0.22	0.80	**0.03**
	4 months	0.16	**0.00**	0.14
Euro/S&P	1 month	0.43	0.76	0.32
	2 months	**0.02**	0.59	**0.04**
	3 months	0.06	0.74	0.13
	4 months	0.37	**0.00**	0.35

lance is used with one, two, three and four months. Figure 5.3 presents the p-values for 10,000 Monte Carlo simulations ($m = 10,000$).

Working with a 95% confidence interval, it is expected to find random points below both the 20 and 5% levels (horizontal lines). However, in December 2006, June 2007 and October 2007, all asset pairs presented at least one horizon below the 5% level. This effect can indicate a presence of a market reaction, a financial contagion, before the crisis announcements and earlier than previous studies about Early Warning Systems (Addo et al. 2013, for example). The p-values for these horizons are presented in Table 5.1.

Having analysed the results for the proposed model, we would like to compare the WVD Space-Time Scan with the Copula approach estimated by filtered conditional residuals obtained from a ARMA/GARCH model as shown by Nystrom and Skoglund (2002). In this case, each univariate time series was modelled as a AR(1)/GARCH(1,1) process, removing both autocorrelation and heterocedasticity present in the original

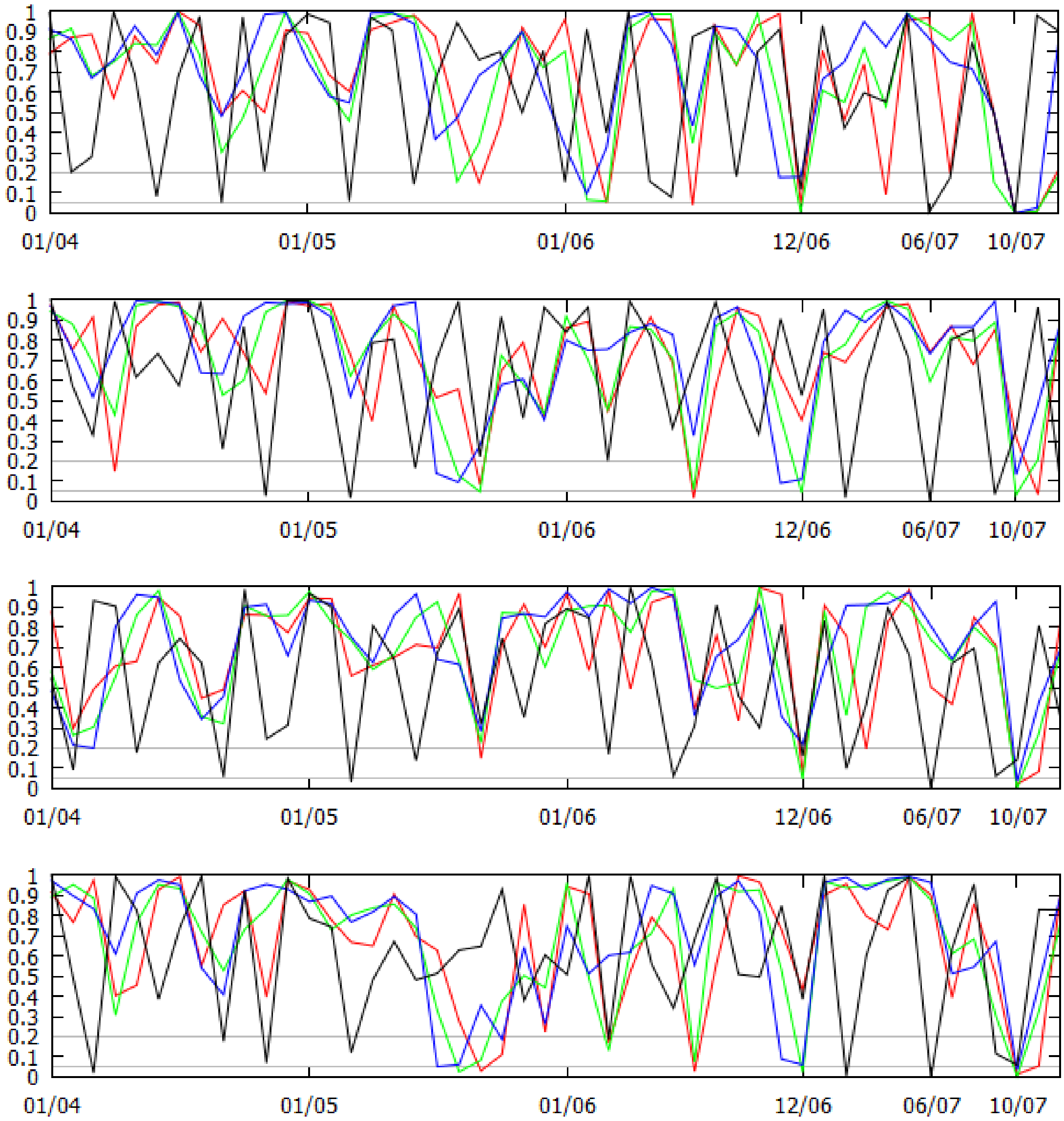


Fig. 5.3 P-values for Corporate Bonds and S&P500, Euro and NASDAQ, Euro and Corporate Bonds, and Euro and S&P500 for 1, 2, 3 and 4 months (red, green, blue and black), respectively.

series. The second step was to estimate the empirical cumulative distribution function (CDF) for each series with a Gaussian kernel to smooth the interior the sample CDF pattern. Each tail, consisting of 15% of the residuals, was associated with a parametric Generalized Pareto Distribution (GDP) and their index (ζ) and scale (β) parameters were estimated optimizing the log-likelihood function. Finally, the standardized residuals are transformed to uniform variates and a t-copula (as described in 2.1) is fitted to the transformed data. Figure 5.4 shows the ρ for the pairs studied.

The horizontal lines in Figure 5.4 show a possible threshold for the coefficient of 0.8 and -0.8. It is worthy mentioning that the interpretation of the correlation coefficient graph is different from the p-values shown in Figure 5.3: while the crisis signal is given by a low p-value level in Figure 5.3, in Figure 5.4 the crisis is signalized by a high correlation coefficient. As can be seen, the positive threshold, i.e. +0.8, was crossed just once, in the first quarter of 2006, by the Corporate Bonds and S&P500 (1-month) series during the period in question. This signal is weaker than the signal generated by the WVD Space-Time Scan procedure.

5.3 International Financial Markets and the WVD Space-Time Scan

In order to analyse the behaviour of the WVD Space-Time Scan for the international markets, equity indexes were used from the US, UK, and Japanese markets: S&P500, FTSE100, and NIKKEI225, respectively. These markets were chosen due to their representativeness in global financial markets and their informational efficiency.

The period in question was from January 2003 until December 2007 for all assets. However, the number of observations are different for each pair of series as only trading days were taken into account. The American and British pair was built with 1250 observations, the American and Japanese series had 1184 observations, and the Japanese and British pair showed 1223 observations.

Similar analysis as described in previous section is made and the p-values are shown in Figure 5.5 and Table 5.2.

As can be seen, in July 2006 the Nikkei/FTSE pair showed two p-values below 5% level, i.e for 2 and 4 months, and the two other pairs in study showed all p-values below 5% level. In December of the same year, both Nikkei/FTSE and FTSE/S&P reached levels below 5% for 2 and 3 month, and Nikkei/S&P reached levels below 5% for all four months. In October 2007, when the crisis was officially declared, all pairs presented p-values below 5% levels.

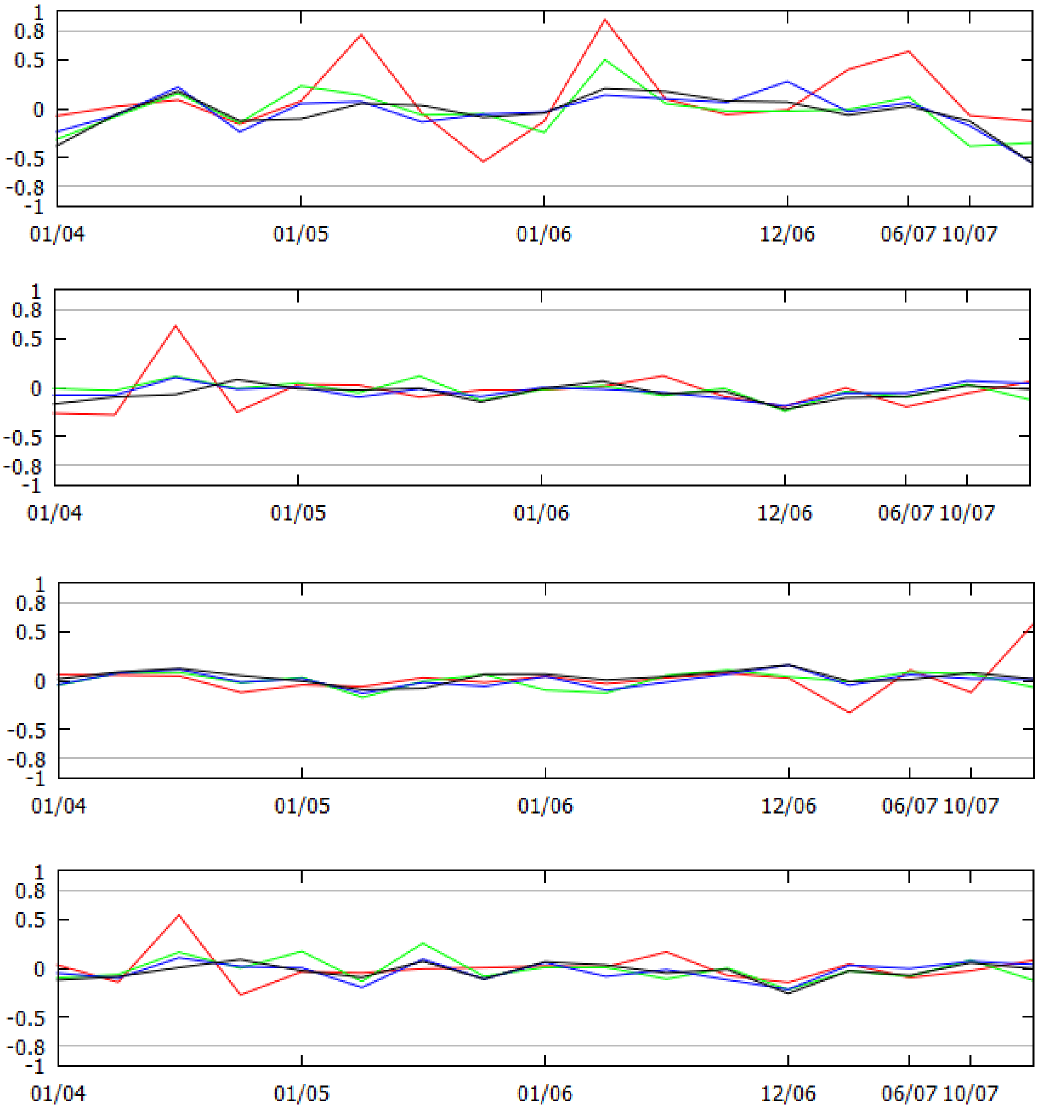


Fig. 5.4 Correlation Coefficients estimated by EVT and Copula for ARMA/GARCH filtered series of Corporate Bonds and S&P500, Euro and NASDAQ, Euro and Corporate Bonds, and Euro and S&P500 for 1, 2, 3 and 4 months (red, green, blue and black), respectively.

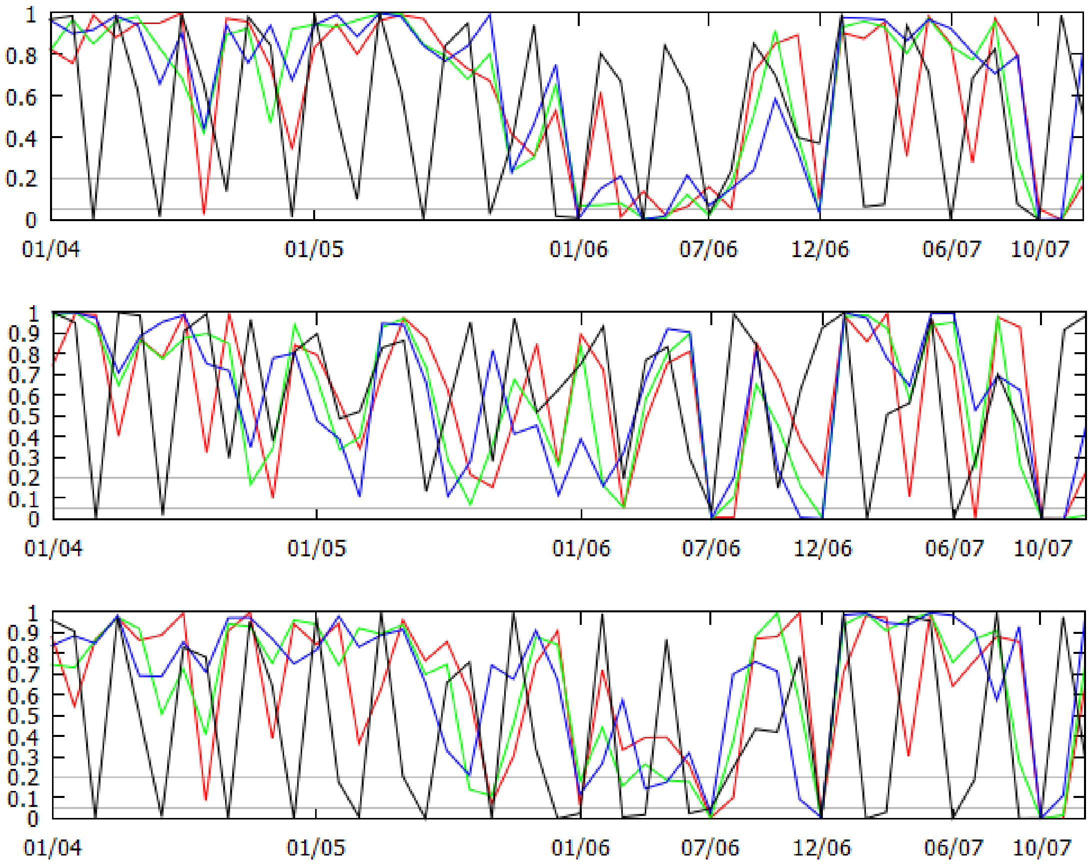


Fig. 5.5 P-values for NIKKEI250 and FTSE100, FTSE100 and S&P500, and NIKKEI250 and S&P500 for 1, 2, 3 and 4 months (red, green, blue and black), respectively.

Table 5.2 P-values for three different dates and four horizons.

	Horizon	07/06	12/06	10/07
Nikkei/FTSE	1 month	0.16	0.10	**0.05**
	2 months	**0.02**	**0.04**	**0.00**
	3 months	0.07	**0.04**	**0.00**
	4 months	**0.03**	0.037	**0.00**
FTSE/S&P	1 month	**0.01**	0.21	**0.00**
	2 months	**0.00**	**0.01**	**0.00**
	3 months	**0.01**	**0.00**	**0.00**
	4 months	**0.04**	0.92	**0.00**
Nikkei/S&P	1 month	**0.00**	**0.01**	**0.00**
	2 months	**0.00**	**0.00**	**0.00**
	3 months	**0.03**	**0.01**	**0.00**
	4 months	**0.05**	**0.00**	**0.00**

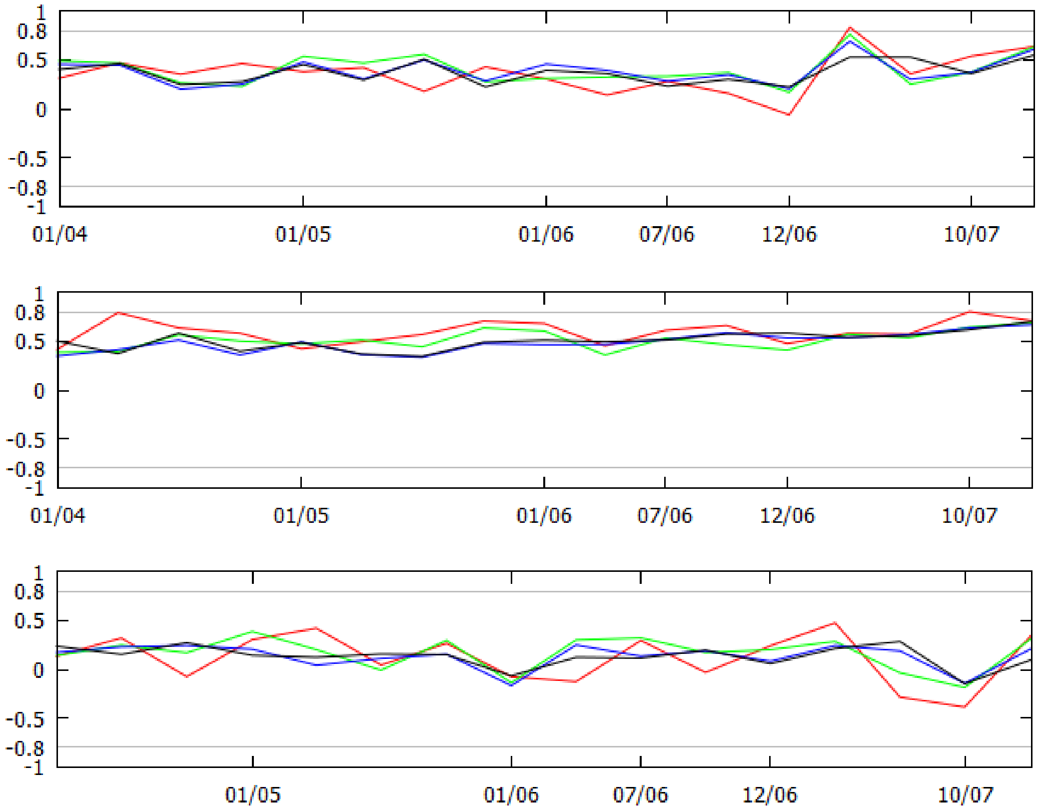


Fig. 5.6 Correlation Coefficients estimated by EVT and Copula for ARMA/GARCH filtered series of NIKKEI250 and FTSE100, FTSE100 and S&P500, and NIKKEI250 and S&P500 for 1, 2, 3 and 4 months (red, green, blue and black), respectively.

However, using the same technique described in section 5.2, i.e. filtering the original series with a AR(1)/GARCH(1,1) process and modelling the each residual series with a Gaussian kernel and GP Distribution before estimating the t-copula function for the dependence, results were not helpful in anticipating the 2007 crisis. The Figure 5.6 displays the behaviour for all three market pairs and for the same window lengths: one, two, three, and four months.

The only signal that crossed the 0.8 correlation threshold was the Nikkei/FTSE 1-month with an increase in all other three periods levels in the first quarter of 2007. Nevertheless, it was not sufficient to raise any flag to signalize any crisis.

Chapter 6

Conclusions and Final Remarks

Dependence modelling has been of major concern in many fields during the last century. So, many measures have been proposed to quantify the relationship between two or more random variables. In this thesis, the Weighted Voronoi Distance (WVD), a new dependence measure based upon spatial tessellation is proposed. This approach uses a well know geometry concept, i.e. the Voronoi diagram, working with the parameters space instead of the geographical points and in a torus instead of the regular Cartesian plane. While not unanimous, such a construction allows users to work with more alternative models and to detect dependence behaviours that are not possible with traditional dependence modelling measure such as linear coefficient or copulas.

Using this new measure, a Space-Time Scan statistics was built to detect the increase in dependence levels recognizing the cluster presence. The method is non-parametric and respects the time structure in dataset to compute the measure which can be an advantage if compared to other methods in applications where assumptions such as independence or autocorrelation cannot be relaxed.

Inference procedure for the WVD Space-Time statistics is presented and Monte Carlo simulations were run to build a thorough cluster detection analysis. Simulations show that this strategy is similar to traditional mehtods in "regular" situations and behave better when the location changes abruptly from north-east to south-west in the map.

Finally, real financial data was used to analyse the detection capacity of the contagion effect in financial markets during the 2007 sub-prime crisis. Two different situations were verified: (i) Different asset classes within the US and (ii) Different countries in international markets. The results from the proposed methodology were compared to Extreme Value Theory and copula approach, and the WVD Space-Time statistics was able to signalize anomalies more strongly than the traditional method.

Thus, it is expected that this tool may help academics, practitioners and regulators to better manage their risk in financial markets.

References

Aas, K. (2004). Modelling the dependence structure of financial assets: A survey of four copulas. *Norwegian Computing Center - NR Note*, SAMBA/22/04.

Acharya, V. and Pedersen, L. (2005). Asset pricing with liquidity risk. *Journal of Financial Economics*, 77:375–410.

Addo, P. M., Billio, M., and Guegan, D. (2013). Nonlinear dynamics and recurrence plots for detecting financial crisis. *The North American Journal of Economics and Finance*, 26:416–435.

Allen, F. and Gale, D. (2000). Financial contagion. *Journal of Political Economy*, 108:1–33.

Andersen, T., Davis, R., Kreiss, J., and Mikosch, T. (2009). *Handbook of Financial Time Series*. Springer-Verlag, Berlin-Heidelberg.

Ang, A. and Bekaert, G. (2002). International asset allocation with regime shifts. *Review of Financial Studies*, 15:1137–1187.

Bae, K., Karolyi, G., and Stulz, R. (2003). A new approach to measuring financial contagion. *The Review of Financial Studies*, 16:717–763.

Bauwens, L., Laurent, S., and Rombouts, J. (2006). Multivariate garch models: A survey. *Journal of Applied Econometrics*, 21:79–109.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31:307–327.

Boubaker, H. and Sghaier, N. (2013). Portfolio optimization in the presence of dependent financial returns with long memory: A copula based approach. *Journal of Banking and Finance*, 37:361–377.

Brunnermeier, M. and Pedersen, L. (2009). Market liquidity and funding liquidity. *Review of Financial Studies*, 22:2201–2238.

Brunnermeier, M. and Perdersen, L. (2005). Predatory trading. *Journal of Finance*, 60:1825–1863.

Calvo, S. and Reinhart, C. (2003). Capital flows to latin america: Is there evidence of contagion effect? *Private Capital Flows to Emerging Markets After the Mexican Crisis*, (G. A. Calvo, M. Goldstein and E. Hochreiter, eds.). Institute for International Economics, Washington, D. C.

Cherubini, Vecchiato, and Luciano (2004). *Copula Models in Finance*. John Wiley & Sons, New York.

Conley, J., Gahegan, M., and Macgill, J. (2005). A genetic approach to detecting clusters in point-data sets. *Geographical Analysis*, 37:286–314.

Deng, X. and Tsui, K. (2013). Penalized covariance matrix estimation using a matrix-logarithm transformation. *Journal of Computational and Graphical Statistics*, 22(2):494–512.

Ding, C., Granger, C., and Engle, R. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, 1:83–106.

Dornbusch, R., Park, Y., and Claessens, S. (2000). Contagion: How it spreads and how it can be stopped. *World Bank Research Observer*, 15:177–197.

Duczmal, L., Cançado, A., and Takahashi, R. (2008). Geographic delineation of disease clusters through multi-objective optimization. *Journal of Computational and Graphical Statistics*, 17(1):243–262.

Duczmal, L., Kulldorf, M., and Huang, L. (2006). Evaluation of spatial scan statistics for irregularly shaped clusters. *Journal of Computational and Graphical Statistics*, 15(2):428–442.

Duczmal, L., Moreira, G., Burgarelli, D., Takahashi, R., Magalhães, F., and Bodevan, E. (2011). Voronoi distance based prospective space-time scans for point data sets: a dengue fever cluster analysis in a southeast brazilian town. *International Journal of Health Geographics*, 10:29.

Dungey, M., Fry, R., González-Hermosillo, B., and Martin, V. (2003). Empirical modelling of contagion: A review of methofologies. *CERF*, 8. Working Paper.

Edwards, S. (1998). Interest rate volatility, contagion and convergence: an empirical investigation of the cases of argentina, chile and mexico. *Journal of Applied Economics*, 1:55–86.

Eichengreen, B., Rose, A., and Wyplosz, C. (1996). Contagious currency crisis. *Scandinavian Economic Review*, 98:463–84.

Engle, R. (1982). Autoregressive conditional heterokedasticity with estimates of the variance of u.k. inflation. *Econometrica*, 50:987–1008.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, 20(3):339–350.

FOMC (2007). Current economic and financial conditions: Summary and outlook. Federal Open Market Committee, Greenbook, Part I, August Meeting. Staff of the Board of Governors of the Federal Reserve System.

Forbes, K. and Rigobon, R. (1999). No contagion, only interdependence: Measuring stock market co-movements. *NBER*, 7267. Working Paper.

Forbes, K. and Rigobon, R. (2002). No contagion, only interdependence: measuring stock market co-movements. *Journal of Finance*, 57:2223–2261.

Frankel, J. and Rose, A. (1996). Currency crashes in emerging markets: an empirical treatment. *Journal of International Economics*, 41:351–366.

Frankel, J. and Saravelos, G. (2010). Are leading indicators of financial crises useful for assessing country vulnerability? evidence from the 2008-09 global crisis. *NBER*, 16047. Working Paper.

Ghrobel, A. and Trabelsi, A. (2009). Measure of financial risk using conditional extreme value copulas with evt margins. *The Journal of Risk*, 11(4):51–85.

Granger, C. and Silvapulle, P. (2001). Large returns, conditional correlation and portfolio diversification: a value-at-risk approach. *Quantitative Finance*, 1:542–551.

Hawkins, J. and Klau, M. (2000). Measuring potential vulnerabilities in emerging market economies. *BIS Working Papers*, 91. Bank for International Settlements.

Huard, D., Évin, G., and Favre, A. (2006). Bayesian copula selection. *Computational Statistics & Data Analysis*, 51(2):809–822.

Kaminsky, G., C., R., and Vegh, C. (2003). The unholy trinity of financial contagion. *Journal of Economic Perspectives*, 17:51–74.

Kaminsky, G., Lizond, S., and Reinhart, C. (1997). Leading indicators of currency crisis. *IMF Staff Papers*, 45.

Kendall, M. (1938). A new measure of rank correlation. *Biometrika*, 30:81–93.

Khalid, A. and Kawai, M. (2003). Was financial market contagion the source of economic crisis in asia? evidence using a multivariate var model. *Journal of Asian Economics*, 14:131–156.

King, M. and Wadhwani, S. (1990). Transmission of volatility between stock markets. *Review of Financial Markets*, 3:5–33.

Kiyotaki, N. and Moore, J. (2002). Evil is the root of all money. *American Economic Review: Papers and Proceedings*, 92:62–66.

Kolev, N. and Paiva, D. (2009). Copula-based regression models: A survey. *Journal of Statistical Planning and Inference*, 139:3847–3856.

Krugman, P. (1979). A model balance-of-payments crises. *Journal of Money, Credit and Banking*, 11:311–25.

Kulldorff, M. (1997). A spatial scan statistic. *Communications in Statistics: Theory and Methods*, 26:1481–1496.

Kulldorff, M. (2001). Prospective time periodic geographical disease surveillance using a scan statistic. *Journal of The Royal Statistical Society Series A*, 164:61–72.

Kyle, A. S. and Xiong, W. (2001). Contagion as a wealth effect. *Journal of Finance*, 56:1401–1440.

Login, F. and Solnik, B. (2001). Extreme correlation of international equity markets. *Journal of Finance*, 56:649–676.

Longstaff, F. (2004). Flight-from-leverage in distressed asset markets. *Journal of Business*, 33:511–526.

Longstaff, F. (2010). The subprime credit crisis and contagion in financial markets. *Journal of Financial Economics*, 97:436–450.

Manner, H. and Reznikova, O. (2012). A survey on time-varying copulas: Specification, simulations, and application. *Econometric Reviews*, 31:654–687.

Meucci, A. (2011). A new breed of copulas for risk and portfolio management. *Risk*, pages 110–114.

Michiels, F. and Schepper, A. (2013). A new graphical tool for copula selection. *Journal of Computational and Graphical Statistics*, 22:471–493.

Naoui, K., N., L., and S., B. (2010). A dynamic conditional correlation analysis of financial contagion: The case of the subprime credit crisis. *International Journal of Economics and Finance*, 2:85–96.

Nelsen, R. B. (2006). *An Introduction do Copulas*. Springer Series in Statistics. Springer, second edition.

Nelson, D. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica*, 51:347–370.

Nystrom, K. and Skoglund, J. (2002). Univariate extreme value theory, garch and measures of risk. *SwedBank*.

Obstfeld, M. (1986). Rational and self-fulfilling balance-of-payments crises. *The American Economic Review*, 76:72–81.

Okabe, A., Boots, B., Sugihara, K., and Chiu, S. (2000). *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. Wiley, UK, second edition.

Pantaleo, E., Tumminello, M., Lillo, F., and Mantegna, R. (2011). When do improved covariance matrix estimators enhance portfolio optimization? an empirical comparative study of nine estimators. *Quantitative Finance*, 11:1067–1080.

Patton, A. (2012). A review of copula models for economic time series. *Journal of Multivariate Analysis*, 110:4–18.

Pearson, K. (1895). Notes on regression and inheritance in the case of two parents. *Proceedings of the Royal Society of London*, 58:240–242.

Pericoli, M. and Sbracia, M. (2003). A primer on financial contagion. *Journal of Economic Surveys*, 17:1139–1180.

Rodriguez, J. (2007). Measuring financial contagion: A copula approach. *Journal of Empirical Finance*, 14:401–423.

Salvadori, G., De Michele, C., Kottegoda, N., and Rosso, R. (2007). *Extremes in Nature*, volume 56 of *Water Science and Technology Library*. Springer.

Silva, R. and Lopes, H. (2008). Copula, marginal distributions and model selection: a bayesian note. *Statistics and Computing*, 18:313–320.

Sklar, A. (1959). Fonctions de répartition à n dimensions et leurs marges. *Publications de l'Institut de Statistique de l'Université de Paris*, 8:229–231.

Spearman, C. (1904). The proof and measurement of association between two things. *The American Journal of Psychology*, 15:72–101.

Spiegelhalter, D., Best, N., Carlin, B., and Van Der Linde, A. (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 64:583–639.

Valdés, R. (1996). *Essays on Capital Flows and Exchange Rates*. PhD thesis, Massachusetts Institute of Technology.

Vayanos, D. (2004). Flight to qaulity, flight to liquidity, and the pricing of risk. *NBER*, 10327. Working Paper.

Wieland, S., Brownstein, J., Berger, B., and Mandl, K. (2007). Density-equalizing euclidean minimum spanning trees for the detection of all disease cluster shapes. *Proceedings of the National Academy of Sciences*, 104:9404–9409.

Yiannakoulias, N., Rosychuk, R., and Hodgson, J. (2007). Adaptations for finding irregularly shaped disease clusters. *International Journal of Health Geographics*, 6.